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Lectures 2

Public Economics

Efficiency (Ref. Myles 1995, ch. 2)

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Introduction

- The competitive equilibrium economy dates back to Walras (1874) and reached its maturity with Debreu (1954): now it is common as *Arrow-Debreu* economy
- This model provides the analytical foundation for the economies analyzed later (benchmark)
- The welfare properties of the economy (*Two Theorems of Welfare Economics*) used as a basis for claims of the competitve outcome (formalization of Smith's (1776) invisible hand)

Introduction

- **First theorem**: a competitve equilibrium is optimal
- **Second theorem**: an optimum can be decentralized as a competitive equilibrium.

They provide the motivation for two alternative viewpoints upon economic policy

a) Policy should always try to move the economy as close to the competitive equilibrium as possible

b) Rationale of what could be achieved if the economy were competitive and a demonstration of why if it cannot, and possibly should not, be achieved in practise (market faliures, poverty)

Outiline of the lecture

- Assumptions
- Walras’ law
- Normalizations and the effect of policy
- Proof of the two theorems and discussion
- Criticism (absence of lump-sum taxes and limitations of Pareto criterion)
- Further on Pareto criterion and social welfare functions

Arrow-Debreu economy

- 1. Two agents: *consumers (households)* and *producers (firms).* The government introduced later
- Households own endowments of goods and shares of firms which yield dividends (no market for the shares). They trade to maximize utility.
- Firms use inputs to produce outputs, subject to technological constraints, to maximize profits

Arrow-Debreu economy

- All trade takes place at a given date at the equilibrium price
- No uncertainty
- All agents are price-takers (this is true if they are infinitesimally small relative to market). In our case this is imposed as an assumption.
- Hence, no monopoly power: agents interact only via price system.
- No public good nor externalities.

Commodities

- *Commodities* are simply defined as the set of goods that are available during the operation of the economy (possibly at future dates)
- Assume that there is a finite number of n commodities (indexed $i=1,\dots n$). Each commodity is defined by its location and time of availability.
- Some of the goods are held as initial endowment
- To each commodity a price is associated (p_i for good i).
- Price can be either in terms of units of a numeraire or of some unit of account (money).
- For the functioning of the economy the interpretation does not really matter (monetary or barter economy): only relative prices determine choices.

Consumers

- Consumers use the income from the sale of the endowment and from dividend payments to purchase (and consume) their preferred choice of commodities.
- The number of consumers is H (index h=1,...H)
- Heach household h has a utility function:

$U^h=U^h(x_1^h,\dots x_n^h)$ (stricly quasi concave; if good i supplied, $x_i<0$)

- **Endowment:**
- $\omega^h=(\omega_1^h,\dots\omega_n^h)$ (included stock of labour services)
- **Shareholdings** of household h in the m firms: $\theta_1^h,\dots\theta_m^h$ *(≥0)*
- So that dividend from heach firm j is $\theta_j^h\pi^j$ *(π is profit)*
- Dividends are all distributed and across households:
- $\Sigma_{h=1\dots H}\theta_j^h=1$*, for all j=1,...m.*

Consumers

- Each household h chooses a vector $(x_1^h, \ldots x_n^h)$

so as to miximise their U, subject to:

- $\Sigma_i p_i\, x_i^h \leq \Sigma_i p_i\, \omega_i^h + \Sigma_j \theta_j^h \pi^j$

- The maximization problem yields individual demand functions:

- $x_i^h = x_i^h(p, \omega^h, \theta^h, \pi)$

and aggregate demand functions (for good i):

$X_i = \Sigma_h x_i^h = X_i(p, \omega, \theta, \pi)$

(some of X possibly negative; existance assumed…)

Producers

- The producers in the economy are the firms that take inputs and turn them into outputs.
- Each firm is characterized by the technology available and aims to maximize profits by their choice of a production plan.
- Each firm, j, is described by its production set, Y^j , which represents the technology of the firm. It is a list of all the alternative combinations of inputs and outputs of which the firm has knowledge.

Production set

Convention in GE theory: measuring inputs as negative numbers and outputs as positive. (inputs subtract to consumption and when aggregation takes place goods that are inputs of firms cancel out with output of others, so that aggregate output is net change in the stock

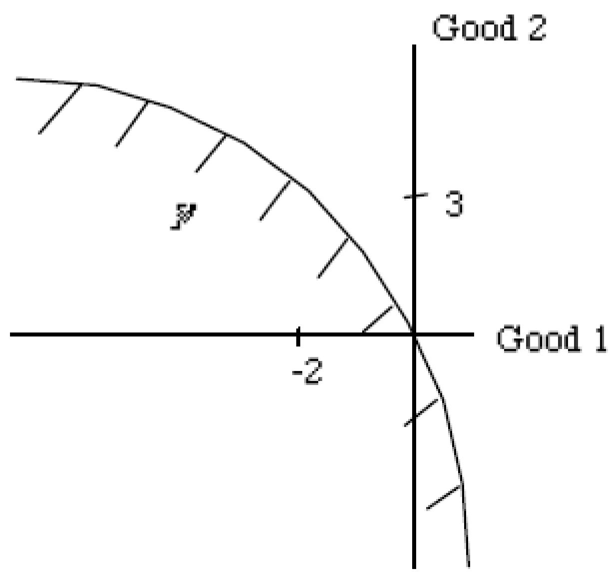


Figure 2.1: A Typical Production Set

Producers

- Structure of production sets: strictly convex set; the origin (inactivity) and the negative orthant (free disposal) are included in the set. no strictly positive vector is in the production set (no free meal).
- In an n-good economy, each firm will choose a production plan y^j, where $y^j \equiv \left(y_1^j, \ldots, y_n^j\right)$
- To maximize profits: $\max_{\{y^j\}} py^j \text{ subject to } y^j \in Y^j$

Example

- Consider the firm shown in Figure 2.1 choosing the production plan described by the vector $y_j = (-2, 3)$. When faced with the price vector $p = (2, 2)$, the firm's level of profit, which is given by the inner product of the price vector and the production vector, is
- $\pi_j = py_j = (2, 2) \cdot (-2, 3) = 2$.

Producers

Profits' maximization yields firm's j supply of each good:

$$y_i^j = y_i^j\left(p\right)$$

and, aggregating over firms we get the **net aggregate supply for i**:

$$Y_i = \sum_{j=1}^{m} y_i^j\left(p\right) = Y_i\left(p\right).$$

Equilibrium: introduction

- Intuitively: the equilibrium of the economy occurs when demands and supplies are in balance: in such a state, each agent is able to carry out its planned action and has no reason for wanting to modify its plan (issues about how the equilibrium is reached: dynamic adjustment? It is a static economy!).
- Since profits can be written as a function of the price vector and eliminating θ and ω (constant parameters)=>Aggregate demand:

$X_i = X_i\ (p, \pi\ (p)) = X_i\ (p)$,

so that we can define the following excess demand function for good i:

- $Z_i\ (p) = X_i\ (p) - Y_i\ (p) - \Sigma_h\ \omega^h_i$
- Definition of equilibrium:
- $Z_i\ (p) \leq 0$, $i = 1, ..., n$ and if $Z_i\ (p) < 0$, $pi = 0$. (2.18)
- In words: equilibrium occurs when excess demand is zero or negative for all goods, with the price of a good being zero if its excess demand is negative.
- Any price vector that satisfies eq. is termed an equilibrium price vector

Walras' law

- Taking excess demand to be less than or equal to zero for the n goods provides n equations in (2.18) to be solved simultaneously. However, the content of Walras' law is that these n equations are not independent and that only $n-1$ actually need to be solved.

Walras' law

- Satisfaction of individual budget constraint implies

$$\sum_{i=1}^{n} p_i x_i^h \leq \sum_{j=1}^{m} \theta_j^h \pi^j + \sum_{i=1}^{n} p_i \omega_i^h,$$

summing over all households gives

$$\sum_{i=1}^{n} p_i X_i \leq \sum_{j=1}^{m} \sum_{h=1}^{H} \theta_j^h \pi^j + \sum_{h=1}^{H} \sum_{i=1}^{n} p_i \omega_i^h,$$

where the first term follows from the definition of aggregate demand.

Typo in Myles 1995

Recalling (2.5) (total shares equal to 1) and π=py we get:

$$\sum_{i=1}^{n} p_i X_i \leq \sum_{i=1}^{n} \sum_{j=1}^{m} p_i y_i^j + \sum_{h=1}^{H} \sum_{i=1}^{n} p_i \omega_i^h.$$

Using the definition of aggregate supply

$$\sum_{i=1}^{n} p_i X_i(p) \leq \sum_{i=1}^{n} p_i Y_i(p) + \sum_{i=1}^{n} p_i \sum_{h=1}^{H} \omega_i^h.$$

or

(2.23) $$\sum_{i=1}^{n} p_i Z_i(p) \leq 0.$$

Walras' law

- The aggregate value of excess demand is nonpositive. Note that it holds for all price vectors, not just equilibrium prices.
- If all households are nonsatiated, so that they spend their entire income, there will be equality in (2.23) and the value of excess demand will be precisely zero.
- Under non-satiation and returning to the equilibrium conditions (2.18), the equality form of Walras' law implies that if $n - 1$ markets have zero excess demand so must the nth.
- Hence there are only $n - 1$ independent equations in (2.18) and it is only necessary to solve for $n - 1$ relative prices.
- Implications: in an economy with a government, if the n markets are in equilibrium and households are meeting their budget constraints, the government must also be meeting its budget constraint (so that is can be disregarded)

Normalizations

- Only relative prices matter in determining demands and supplies; this implies that there is a degree of freedom in the measurement of prices since the scale of prices does not matter.
- In order to remove this freedom is invariably prices are restricted to belong to a compact set that is capable of capturing all feasible price ratios. The most commonly used compact sets are the simplex, so that the prices must satisfy: $\sum_{i=1}^{n} p_i = 1,$
- Convention: take 1 price as numeraire.
- As for taxation, select a good as numeraire, fix its price at 1 and tax rate at zero. Other prices and tax rates measured relative to this good

Core of the economy

- Core: stems from considering economic activity as a cooperative game
- Consider an exchange economy (so there is no production). Each of the H households has an initial endowment of the n goods.
- Assume that the households form coalitions (rather than conducting bilateral exchanges) and allocate the total endowment of each coalition amongst the members of that coalition. A coalition can therefore be composed of between 1 and H households. At any instant of time a household belongs to only one coalition.
- Given some allocation for the economy $\{x^1,\ldots,x^H\}$ coalition S can improve upon this allocation if there is some allocation $\{\hat{x}^h\}$ for $h \in S$ such that

(i) $\sum_{h \in S} \hat{x}^h = \sum_{h \in S} \omega^h$;
(ii) $\hat{x}^h$ is preferred to x^h by all h.

- If this allocation exists, x is not accepted by S, and the process would continue until an allocation is reached that cannot be improved upon.
- The core of the economy is defined as the set of allocations which cannot be improved upon by any coalition. Does this set of allocations exists?

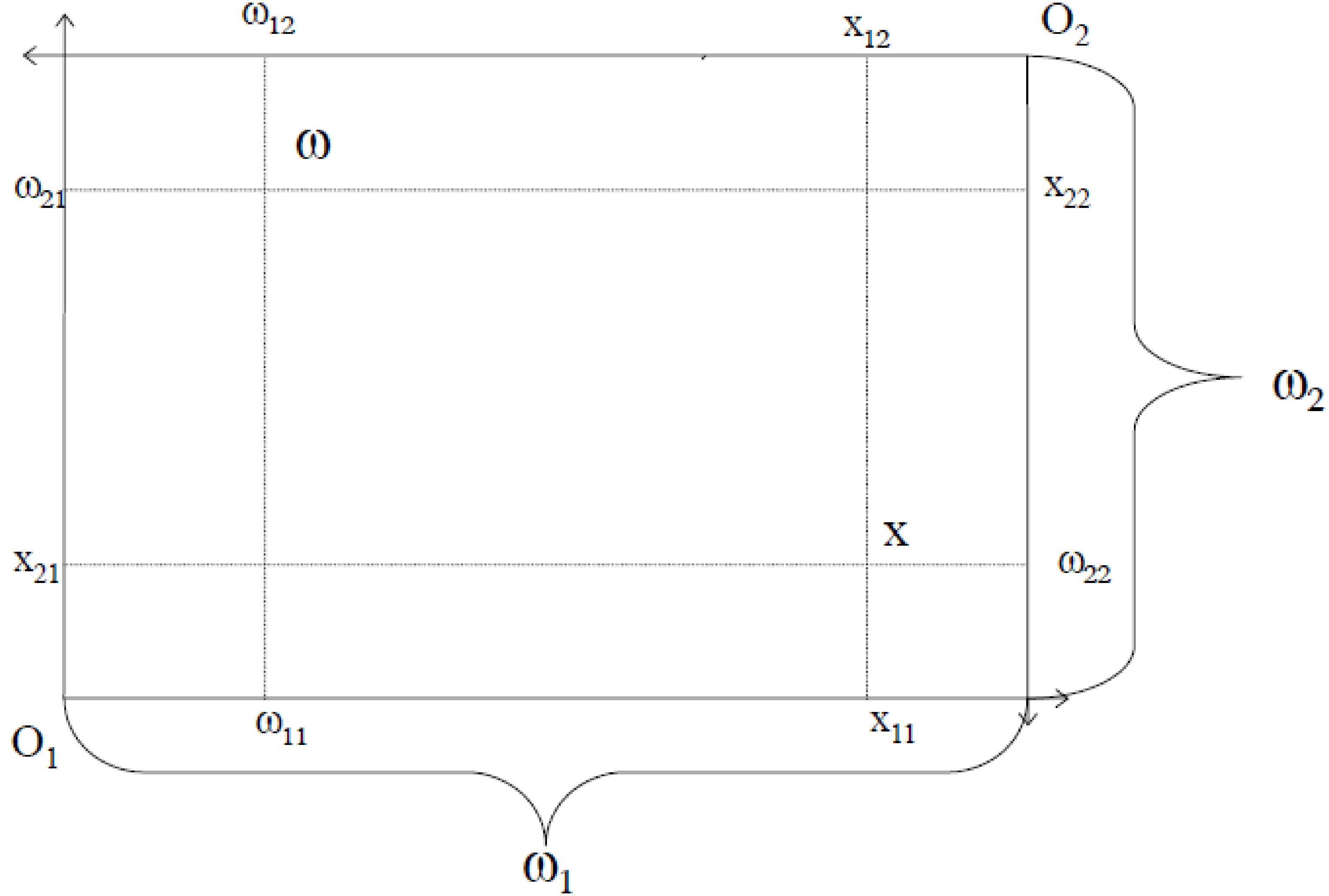
Core of the economy

Theorem 1 *If* $\{\tilde{x}^h\}$ *is the equilibrium allocation for an Arrow-Debreu economy with endowments* $\{\omega^h\}$ *and* $\tilde{p}$ *the corresponding equilibrium price vector, then* $\{\tilde{x}^h\}$ *is in the core of the economy.*

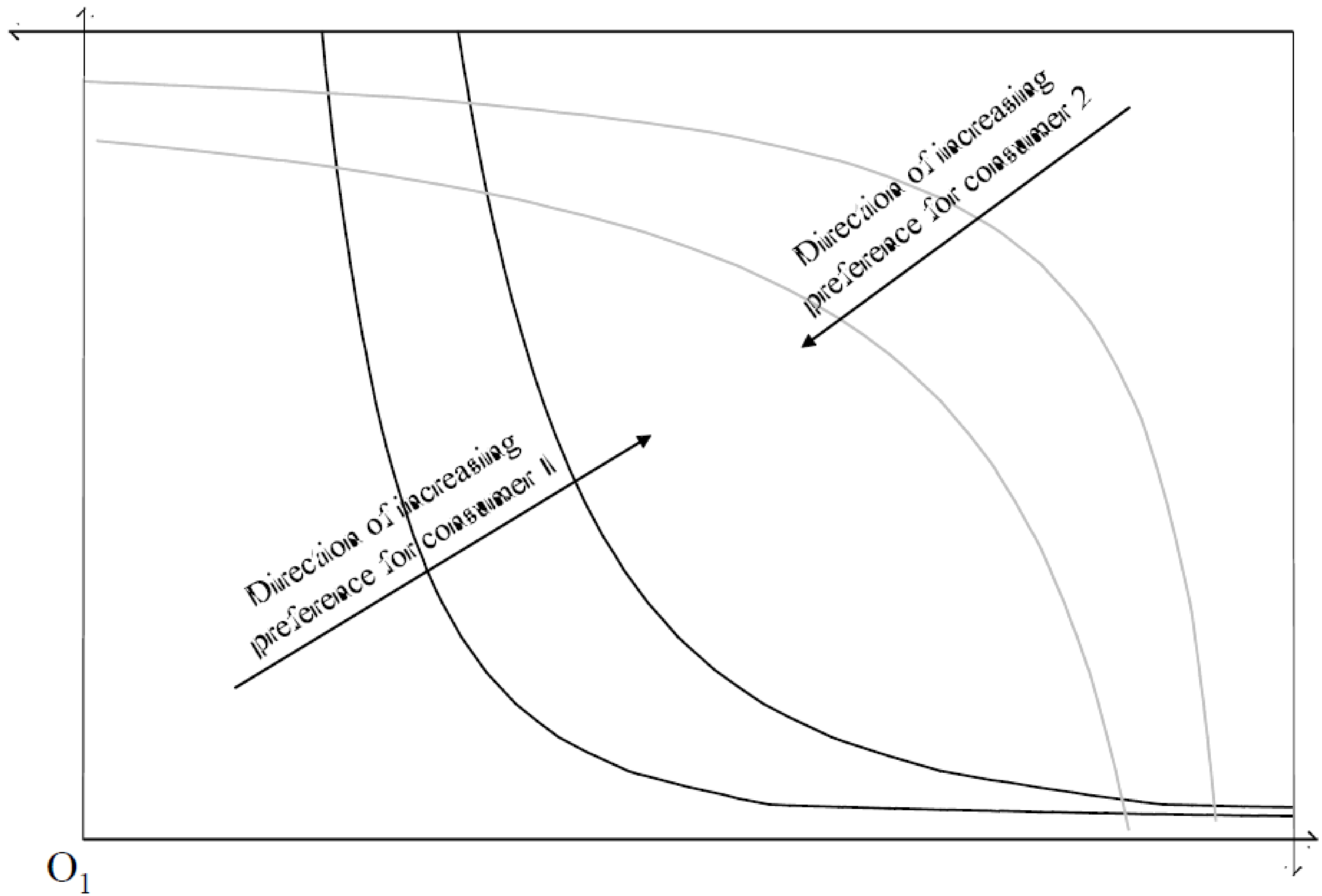
Proof. Assume that the claim is not true so that the exists some coalition S that can improve upon the allocation $\{\tilde{x}^h\}$ with allocation $\{\hat{x}^h\}$, $h \in S$. This implies, from (i) that $\sum_{h\in S} \hat{x}^h = \sum_{h\in S} \omega^h$. However, since $\{\tilde{x}^h\}$ were the optimal choices for the households in the economy at prices $\tilde{p}$, condition (ii) implies $\tilde{p}\hat{x}^h > \tilde{p}\omega^h$ for all $h \in S$. Summing this over $h \in S$ gives

$\tilde{p}\sum_{h\in S} \hat{x}^h$ > $\tilde{p}\sum_{h\in S} \omega^h$. This contradicts feasibility and proves the theorem. || ■

The Edgeworth box (2 individuals-2 goods)

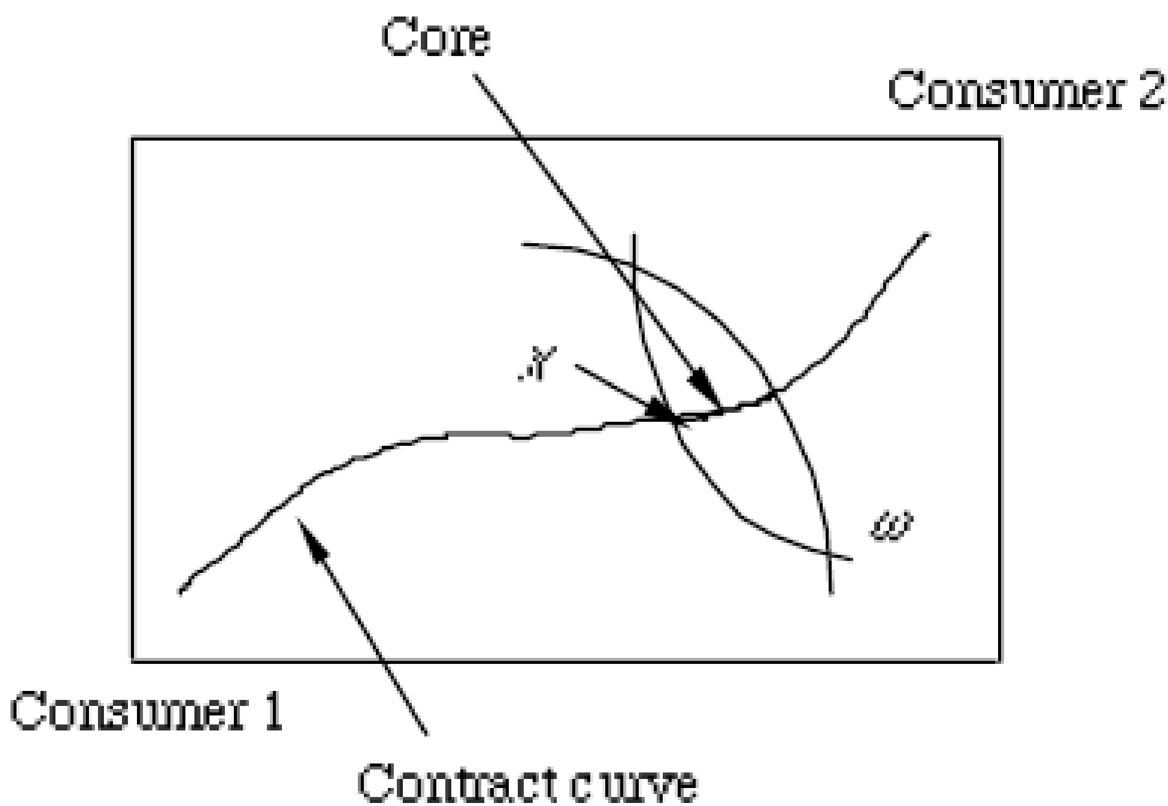


The Edgeworth box (2 individuals)



The core

The core of such an economy is that part of the contract curve which is in the trading set and the competitive equilibria are generally a subset of this



Theorem 2 (Debreu/Scarf)

- Introducing additional households into a two-household economy makes possible the formation of new coalitions: some coalitions will be able to improve upon allocations that are in the core of the two-household economy
- **CORE CONVERGENCE THEOREM**
- *Consider an exchange economy with m types of household and r households of each type. Assume the preferences of each type of household satisfy insatiability (given any consumption plan xh there exists* $\tilde{x}_h$ *strictly preferred to* x_h*), strong convexity and continuity and that the endowment of each household is strictly positive. Then, if an allocation is in the core for all values of r, it is a competitive equilibrium allocation.*
- Proof: See Debreu-Scarf (1963)
- Theorem says that as the size of the economy increases by replication, the core of the economy shrinks to the set of competitive equilibria
- Also without "replica", as shown by Aumann, (1964), for a continuum economy the core and the set of competitive equilibria are identical.

Welfare properties of the competitve equilibrium (Debreu 1954)

Definition 3 *Feasibility*

An array of consumption vectors $\{x^1, ..., x^H\}$ *is feasible if* $x^h \in X^h$, *all* h, *and there exists an array of production vectors* $\{y^1, ..., y^m\}$, *each* $y^j \in Y^j$, *such that*

$$x \leq y + \omega, \tag{2.28}$$

where

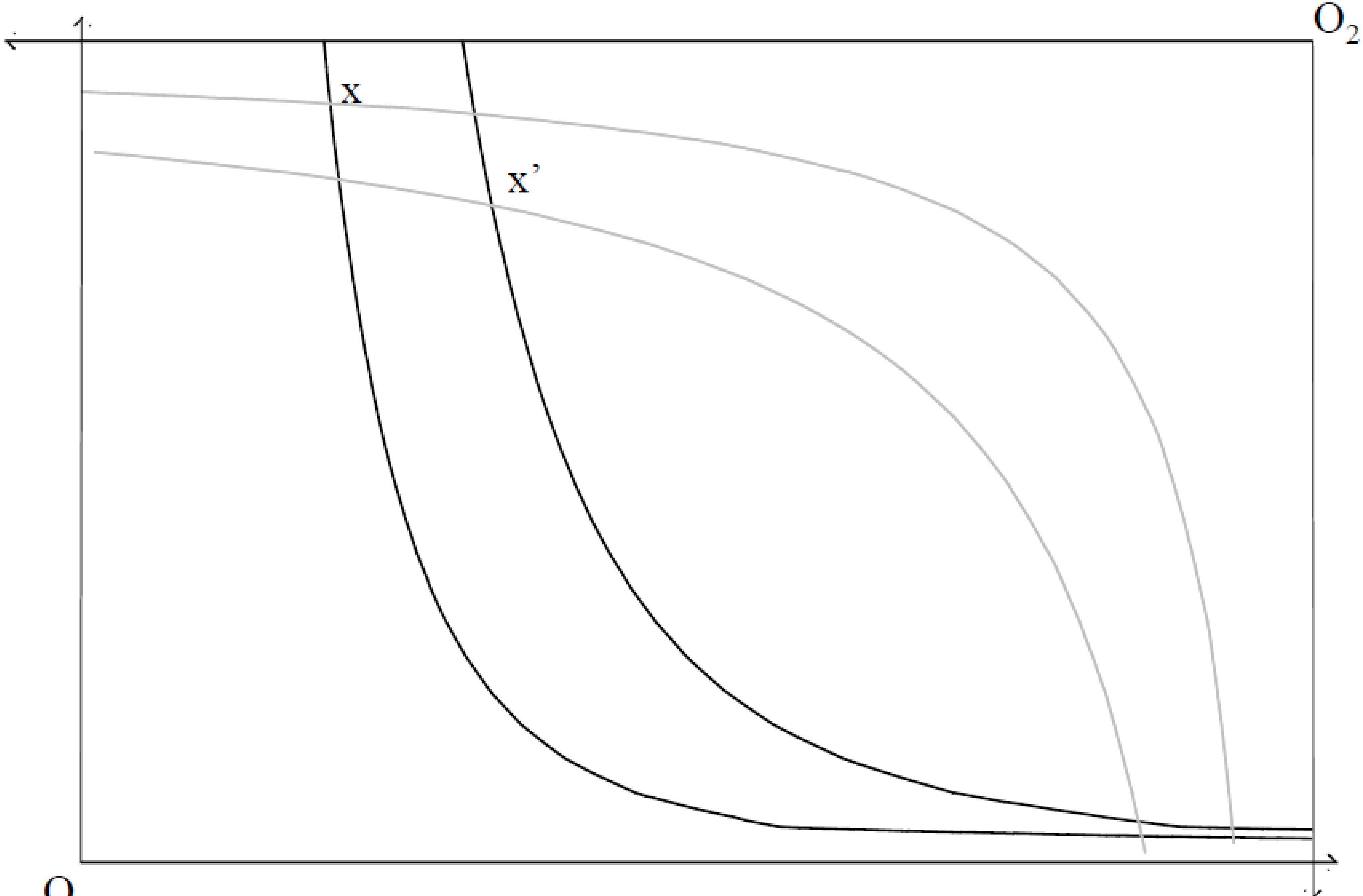
$$x = \sum_{h=1}^{H} x^h, y = \sum_{j=1}^{m} y^j, \omega = \sum_{h=1}^{H} \omega^h. \tag{2.29}$$

Definition 4 *Pareto Optimality (P.O.) A feasible consumption array* $\{\hat{x}^h\}$ *is Pareto optimal if there does not exist a feasible array* $\{\bar{x}^h\}$*such that*

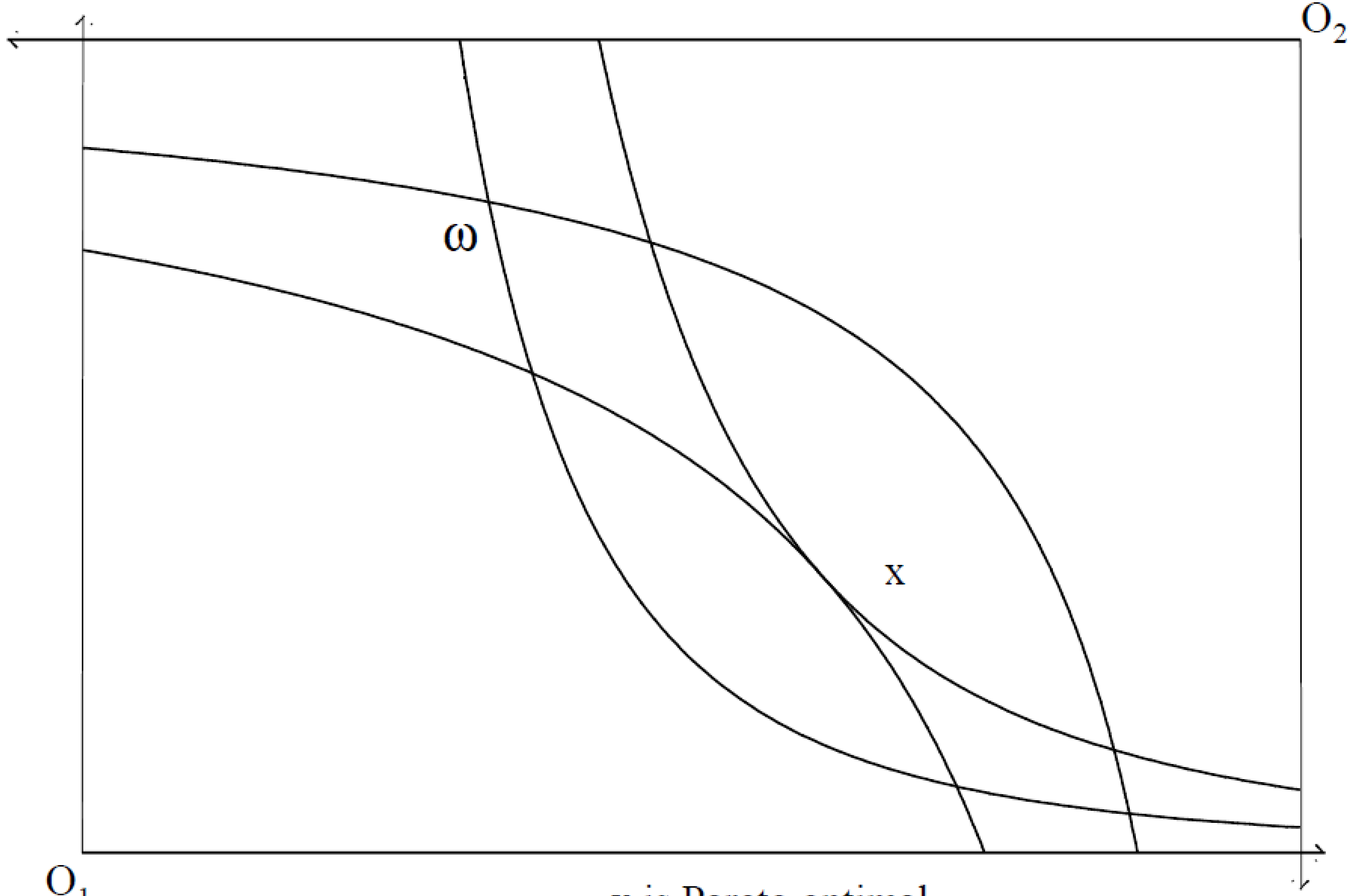
$$U^h\left(\bar{x}^h\right) \geq U^h\left(\hat{x}^h\right), h = 1, ..., H, \tag{2.30}$$

with

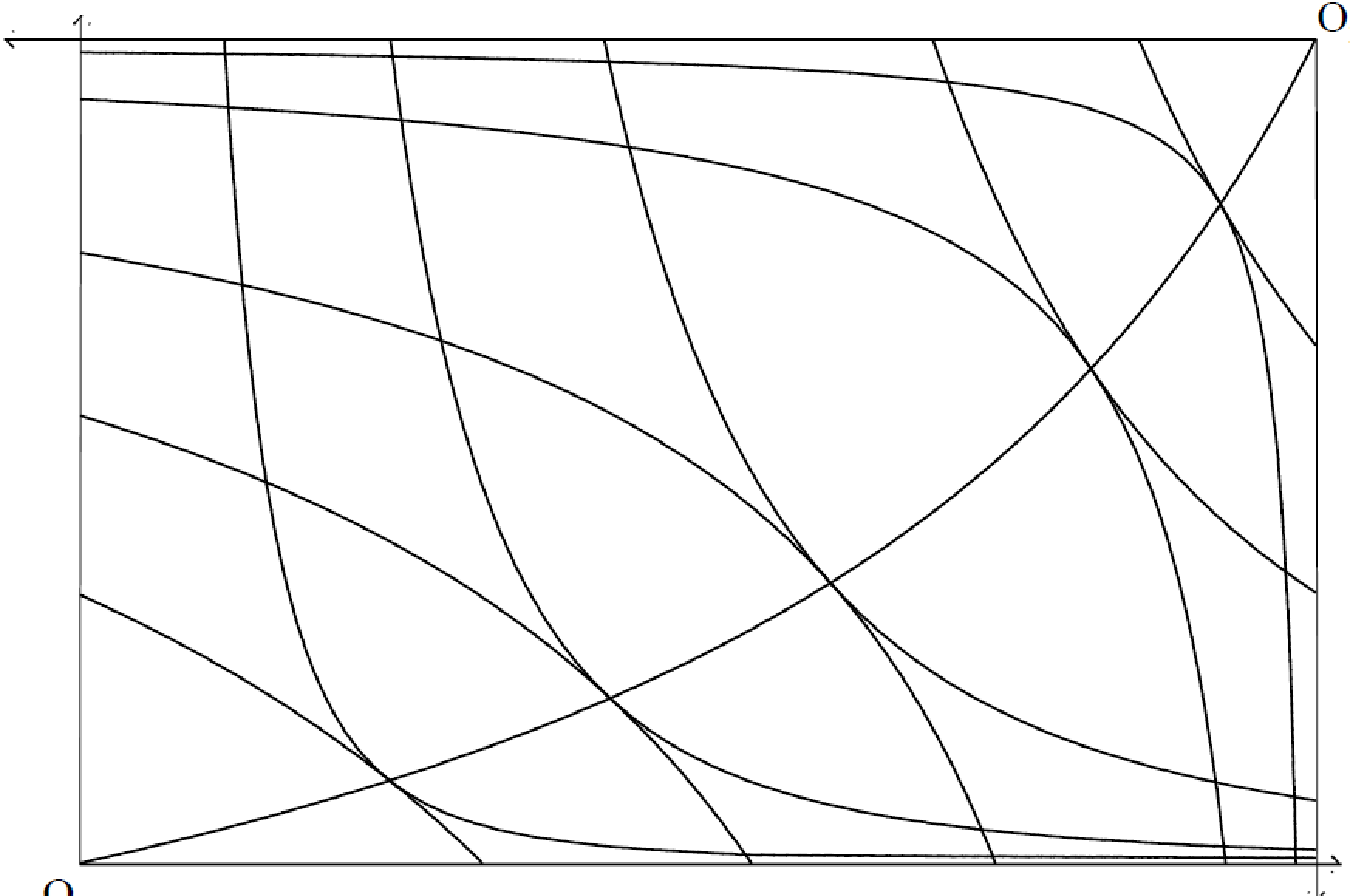
$U^h\left(\bar{x}^h\right) > U^h\left(\hat{x}^h\right)$, *for at least one* h.



x and x' are not Pareto optimal



x is Pareto optimal



The locus of all Pareto optimal allocations (contract curve)

Definition of Competitive equilibrium

Definition 5 *Competitive Equilibrium (C.E.) An array* $\left[p, \{\hat{x}^h\}, \{\widehat{y}^j\}\right]$ *is a competitive equilibrium if*

$$\hat{x}^h \in X^h, \hat{p}\hat{x}^h \le \hat{p}\omega^h + \sum_{j=1}^{m} \theta_j^h \hat{p}\widehat{y}^j, h = 1, ..., H, \tag{2.32}$$

$$\widehat{y}^j \in Y^j, j = 1, ..., m, \tag{2.33}$$

and
(i) $U^h\left(\hat{x}^h\right) \ge U^h\left(x^h\right)$ *for all* $x^h \in X^h$ *such that* $\hat{p}x^h \le \hat{p}\omega^h + \sum_{j=1}^{m} \theta_j^h \hat{p}\widehat{y}^j$,
(ii) $\hat{p}\widehat{y}^j \ge \hat{p}y^j$, *all* $y^j \in Y^j$,
(iii) $\hat{x} \le \widehat{y} + \omega$.