Public Economics

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Market failures: Public goods

First part

References

- Chapter 9 Myles: sections, 1,2,3 (3.1, 3.4), 4.1, 6 (6.1), 6, 7 (7.1, 7.2, up to p. 299)
- Chapter 16 Atkinson-Stiglitz

Definitions of Public Goods

- Non excludability: if the public good is supplied, no household can be excluded from consuming it except, possibly, at infinite cost;
- Non rivalry: Consumption of the public good by one household does not reduce the quantity available for consumption by any other;
- Examples: defense system, lighthouse (however transmission of a TV signal can be excludable)
- Impure PG and congestion (parks, roads can be rivalrous)

Implications

- Non-excludability: no price system to control consumption; no household can be prevented to consume once the PG is produced. Concept of competitive economy does not apply, nor First and Second Theorems of welfare do apply.
- Non-rivalry: all households (can) consume simultaneously the level of PG that is produced. (however, also free disposal can be dealt with)

Optimal provision

- 1 single PG available, G, with no disposal
- H households, indexed h=1…H with

$U^h=U^h(x^h,G)$

x^h is the vector of private goods (i=1…n).

G is a pure PG.

Implicit production set:

$F(X,G)\leq 0$

And $X=\Sigma_h x^h$

First best or Pareto efficient allocations (Samuelson 1954)

- The government maximizes the utility level of the first household, s.t. households 2 to H obtain given utility levels and feasibility
- varying utility levels of 2 to H traces out the set of Pareto-efficient allocations

Solution

- $L=U^1(x^1,G)+\Sigma_{h=2}\,\mu^h[U^h(x^h,G)-\,U^h\_\,]-\lambda F(X,G)$

(as an alternative and equivalent way):

$L=\Psi(U^1(x^1,G),\,U^2(x^2,G),\ldots,\,U^H(x^H,G))\,-\lambda F(X,G)$

Maximization w.r.t. to component x_i^h of x

$\partial L/\partial x_i^h=\mu^h(\partial U^h/\partial x_i^h)-\lambda(\partial F/\partial X_i)=0\quad h=1,\ldots,H,$ with $\mu^1=1$ for h=1 and $\forall$ *i. (9.5)*
Max w.r.t. G

- $\partial L/\partial G=\,\Sigma_{h=1\ldots H}\,\mu^h(\partial U^h/\partial G)-\lambda(\partial F/\partial G)=0$ (9.6)

- *Solving for* μ^h in (9.5) and subs into (9.6):

Solution

$\Sigma_{h=1\ldots H}\,[(\partial U^h/\partial G)/(\partial U^h/\partial x_i^{\,h})]=(\partial F/\partial G)/(\partial F/\partial X_i)$

$i=1,\ldots,n.$

That is:

$\Sigma_{h=1\ldots H}(MRS_{Gi}{}^{h})=MRT_{Gi}$

Samuelson's rule: Pareto-efficient provision of the PG occurs when the MRT between PG and Private G is equated to the sum, overall all households, of the MRS.

Comments

- Optimal provision of Private goods:
- $MRS_{ji}^{h}=MRT_{ji}$
- An extra unit of PG increases the utility of all households, so that the social benefit is the sum of all marginal benefits (MRSs). At the optimum this must equate the social marginal cost (MRT)
- As for Priv-Gs, an extra unit of a Pr good only increases the welfare of its single recipient and at the optimum MB are equalized across households and to MC.

Public input

- Derive the efficiency conditions for the supply of a pure public input.
- Consider an economy with m firms each using labour and the public good to produce a single form of output.
- Denoting the labour use of firm j by l^j, the firm's production function is given by

$$y_j = f^j\left(\ell^j, G\right). \tag{9.17}$$

The public good is produced by using labour alone according to the production function $G = \phi\left(\ell^G\right)$. This is assumed to have an inverse $\ell^G = \Theta\left(G\right), \Theta\left(G\right) \equiv$

$\phi^{-1}(G)$. The equilibrium conditions, that supply must equal demand for goods and labour, are given by

$$\sum_{h=1}^{H} x^h = \sum_{j=1}^{m} y^j = \sum_{j=1}^{m} f^j\left(\ell^j, G\right), \tag{9.18}$$

and

$$\sum_{h=1}^{H} \ell^h = \sum_{j=1}^{m} \ell^j + \Theta(G). \tag{9.19}$$

For this economy, an optimum allocation is found from the Lagrangean

$$\begin{aligned} L \;=\; & U^1\left(x^1, \ell^1\right) + \sum_{h=2}^{H} \mu^h \left[U^h\left(x^h, \ell^h\right) - \overline{U}^h\right] + \lambda \left[\sum_{h=1}^{H} x^h - \sum_{j=1}^{m} f^j\left(\ell^j, G\right)\right] \\ & + \rho \left[\sum_{h=1}^{H} \ell^h - \sum_{j=1}^{m} \ell^j - \Theta(G)\right]. \end{aligned} \tag{9.20}$$

Typo in
Myles

Carrying out the maximisation in (9.20) provides the efficiency criteria

$$\frac{\frac{\partial U^h}{\partial \ell^h}}{\frac{\partial U^h}{\partial x^h}} = -\frac{\partial f^j}{\partial \ell^j}, \quad h = 1, ..., H \text{ and } j = 1, ..., m, \tag{9.21}$$

and

$$\sum_{j=1}^{m} \frac{\partial f^j}{\partial G} = \frac{\partial f^j}{\partial \ell^j}\Theta', \quad j = 1, ..., m. \tag{9.22}$$

The first condition, (9.21), ensures that the marginal rate of substitution between labour and consumption is equated between households and this value is set equal to (minus) the firms' common marginal product of labour. This is a standard efficiency condition. Condition (9.22) is the form of the Samuelson rule for the public input and requires the sum of marginal products of the public inputs for the firms to equal the private good foregone in producing marginally more public good.

Comments

- This completes the analysis of rules for efficient provision of PG
- The Samuelson rule may characterise the set of Pareto efficient outcomes but it cannot in general be implemented.
- This motivates the study of feasible allocation mechanisms and the comparison of their outcomes to those that satisfy the Samuelson Rule.

Personalised prices and the Lindhal equilibrium (4.1.) p. 272

- After providing the rule for Pareto efficient provision of a PG, the natural question is whether there is any form of economy in which competitive behaviour will lead to an efficient outcome.
- The equilibrium in the standard model of Chapter 2 will not be Pareto efficient in the presence of public goods.
- This arises from the fact that consumers differ in the valuation they place upon a given supply of the public good.
- Insisting that they all pay an identical price for the supply cannot therefore be efficient.

Cont’d

- Following this reasoning, it would appear likely that Pareto efficiency would result if each consumer could pay an individual or personalised price for the good. In this way, each will be paying a price that reflects their valuation.
- Allowing such personalised prices represents an extension of the Arrow-Debreu economy which assumed that each commodity had a single price.
- The equilibrium with personalised prices is often called a Lindahl equilibrium after its introduction by Lindahl (1919).

Lindhal equilbrium: assumptions

- Consider an economy with 2 households who have an endowment of ω^h units, $h = 1, 2$, of the numeraire which they supply inelastically to the market. Each household therefore has a fixed income of ω^h.
- There is a single private good produced with constant returns to scale using the numeraire alone and the units of measurement of this good are chosen so that a unit of output requires one unit of numeraire input.
- The price of the private good is therefore also equal to one.
- Production of the public good is subject to constant returns to scale and each unit requires p_G units of labour.
- The marginal rate of transformation in production between the public good and the private good is therefore constant at p_G.

The model

- Assume that each household has a utility function such that

- $U^h = U^h(x^h, G^h)$, $h = 1, 2$, (9.23)

- where x^h is the quantity consumed of the single private good and G is the quantity of the public good. Utility is non-decreasing in x^h and G. Now let G^h denote the quantity of the public good that household h would like to see provided when faced with the budget constraint:

- $x^h + \tau^h p_G G^h = \omega^h$. (9.24)

The model

- In (9.24) $p_G G^h$ is the total cost of providing the good and τ^h the fraction of this paid by h. From (9.23) and (9.24) household h chooses G^h to maximise
- $U^h = U^h(\omega^h - \tau^h p_G G^h, G^h)$ (9.25)
- Maximisation with respect to G^h yields
- $U^h_G / U^h_x = \tau^h p_G$. (9.26)
- Solving (9.26) using (9.24) for G^h generates the Lindahl reaction function
- $G^h = L^h(\tau^h; \omega^h)$ (9.27)
- Demand functions: Household's demand for the public good as a function of the cost share it faces and its initial endowment. If the second-order condition for maximising (9.25) is satisfied and the utility function is strictly concave, then $L^h(\cdot)$ is a decreasing function of τ^h. (prove it as an exercise)

Lindhal equilibrium: definition

A *Lindahl equilibrium* is a pair of cost shares $\left\{\hat{\tau}^1, \hat{\tau}^2\right\}$ such that

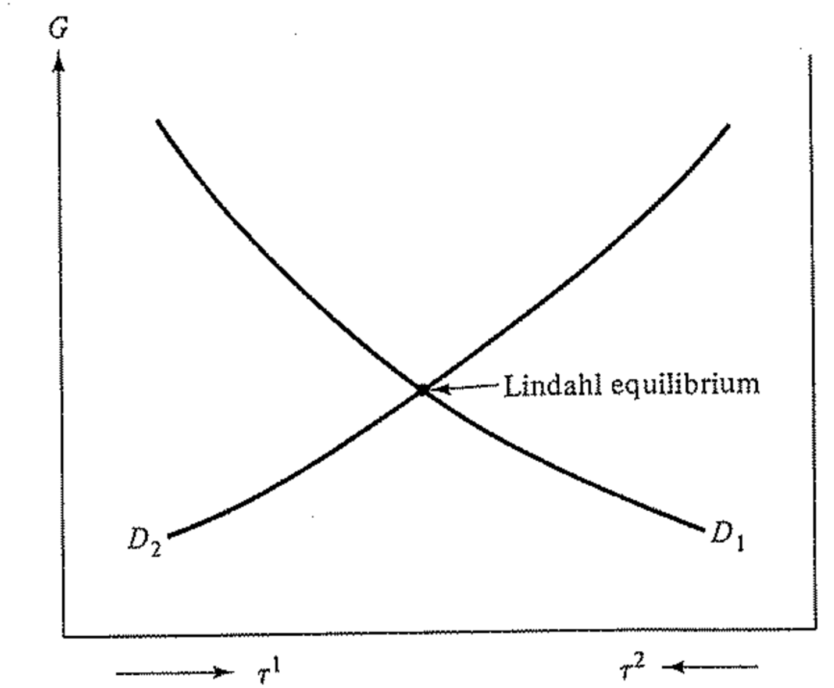
(i) $\hat{\tau}^1 + \hat{\tau}^2 = 1,$

and

(ii) $L^h\left(\hat{\tau}^h; \omega^h\right) = G^* \geq 0, \; h = 1, 2.$

The first condition guarantees that sufficient revenue will be obtained to finance the equilibrium public good provision and the second-condition that the households will both be satisfied with the supply. It follows from the fact that utility is non-decreasing in G that the cost shares will be non-negative.

Lindhal equilibrium



The Lindhal equilibrium is given by the intersection of the reaction functions. the equilibrium is Pareto efficient.

To demonstrate the latter point, note that (9.26) must hold for both households at the equilibrium. Summing for the two households then gives

$$\sum_{h=1}^{2} \frac{U_G^h}{U_x^h} = \sum_{h=1}^{2} MRS_{Gx}^h = \sum_{h=1}^{2} \tau^h p_G = p_G = MRT_{Gx}. \qquad (9.28)$$

Conclusions & extensions

- Since (9.28) is the Samuelson rule for this economy, it demonstrates that the Lindahl equilibrium is Pareto efficient.
- This establishes a form of the First Theorem of Welfare Economics for the Lindahl equilibrium.
- The relation of the Lindahl equilibrium to the Second Theorem will not be investigated.
- (It is sufficient to note that by redistributing the initial endowment it is possible to generate a new Lindahl equilibrium which represents another point in the set of Pareto efficient outcomes).
- Lindhal equilibrium is in the core of the economy
- Private provision of public goods is in general not efficient