Public Economics

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Market failures: Public goods

Part two

Finance by taxation

- Assume lump sum taxes are not feasible and only taxes on commodities are available
- The aim of the analysis is to determine how the distortions caused by the commodities taxes affect the Samuelson Rule and the level of provision. This is undertaken by following the work of Atkinson and Stern (1974).

Distortionary taxes with identical households

Each of the identical consumers maximises their utility $U(x, G)$ subject to the budget constraint $qx = 0$ where q is the vector of post-tax prices and x the vector of net demands. There is one change from the standard commodity tax model: revenue must now equal expenditure, G, on the public good

$$H \sum_{i=1}^{n} t_i x_i = G. \tag{9.44}$$

Market clearing implies that the revenue constraint and the production constraint may be used interchangeably as argued in Chapter 2 above. The production constraint is used and is written in the form $F(X, G) = F(Hx, G) = 0$. It is assumed that

input

$$F_1 = \frac{\partial F}{\partial X_1}, \tag{9.45}$$

and good 1 is taken to be the numeraire with $q_1 = p_1 = 1$. Pre-tax prices are chosen so that $F_k = p_k$.

For the choice of optimal tax rates, t, and quantity of public good, the appropriate Lagrangean is

V=Indirect utility function after individual maximization

Aggregate Demand functions after maximization

$$L = HV(q, G) - \lambda F(X(q, G), G). \tag{9.46}$$

From this, the first-order condition for the choice of G is

$$\frac{\partial L}{\partial G} \equiv H\frac{\partial V}{\partial G} - \lambda\left[\sum_{i=1}^{n} F_i \frac{\partial X_i}{\partial G} + F_G\right] = 0, \tag{9.47}$$

which, using the definition of pre-tax prices, can be written

I= lump sum income

$\frac{\partial V}{\partial I}$

$$H\frac{\frac{\partial V}{\partial G}}{\alpha q_k} = \frac{p_k}{q_k}\frac{\lambda}{\alpha}\frac{F_G}{F_k} + \frac{\lambda}{\alpha q_k}\sum_{i=1}^{n} p_i \frac{\partial X_i}{\partial G}, \tag{9.48}$$

where α is the marginal utility of income for each consumer. From each consumer's first-order condition for the utility maximising choice of good k, $\frac{\partial U}{\partial x_k} = \alpha q_k$. Therefore the term

$$H\frac{\frac{\partial V}{\partial G}}{\alpha q_k} = H\frac{\frac{\partial U}{\partial G}}{\frac{\partial U}{\partial x_k}}, \tag{9.49}$$

is the sum of marginal rates of substitution between the public good and private good k.

Deviation from optimality (second best)

Evaluating the first-order condition (9.48) for $k = 1$, so that $q_1 = 1$, it can be rearranged to give

$$\frac{F_G}{F_1} = \frac{\alpha}{\lambda} H \frac{\frac{\partial U}{\partial G}}{\frac{\partial U}{\partial x_k}} + \sum_{i=1}^{n} [q_i - t_i] \frac{\partial X_i}{\partial G}, \tag{9.50}$$

but since the consumers' budget constraints imply $\sum_{i=1}^{n} q_i \frac{\partial X_i}{\partial G} = 0$, (9.50) can be written

$$\frac{F_G}{F_1} = \frac{\alpha}{\lambda} H \frac{\frac{\partial U}{\partial G}}{\frac{\partial U}{\partial x_k}} - \frac{\partial \sum_{i=1}^{n} t_i X_i}{\partial G}. \tag{9.51}$$

Typo in Myles

MRT="social marginal cost"

ΣMRS= Social-private Mg Benefit

Revenue effect of G: depends on the degree of complementarity of G with X_i

In general we cannot state if provision of G is lower than the first best

Note that alpha=individual marginal utility of income; lambda=social marginal utility of income…equal only with lump sum taxes. In fact:

To isolate the first effect assume that $\frac{\partial \sum_{i=1}^{n} t_i X_i}{\partial G} = 0$ so that the public good is revenue neutral. In this case the departure from the first-best is determined by $\frac{\alpha}{\lambda}$ alone. To proceed further, consider the choice of tax rate for good k. From the Lagrangean

$$\frac{\partial x_i}{\partial q_k} \equiv \frac{\partial x_i}{\partial t_k},$$

$$H\frac{\partial V}{\partial q_k} = \lambda \sum_{i=1}^{n} F_i \frac{\partial X_i}{\partial q_k} = \lambda \frac{\partial \sum_{i=1}^{n} p_i X_i}{\partial t_k}. \tag{9.52}$$

Using Roy's identity and the fact that

$$\frac{\partial \sum_{i=1}^{n} p_i X_i}{\partial t_k} + \frac{\partial \sum_{i=1}^{n} t_i X_i}{\partial t_k} = \frac{\partial \sum_{i=1}^{n} q_i X_i}{\partial t_k}, \tag{9.53}$$

=0 from differentiating individual budget constraint

(9.51) can be written

$$\frac{\alpha}{\lambda} = \frac{\frac{\partial \sum_{i=1}^{n} t_i X_i}{\partial t_k}}{X_k}. \tag{9.54}$$

Different from 1

From Roy's identity it follows that

$$\frac{\partial V}{\partial q_k} = -\frac{\partial V}{\partial I} x_k = -\alpha x_k, \tag{4.10}$$

Finally, using the Slutsky equation,
$$\frac{\partial x_i}{\partial q_k} = S_{ik} - x_k \frac{\partial x_i}{\partial I}.$$

and $\dfrac{\partial x_i}{\partial q_k} \equiv \dfrac{\partial x_i}{\partial t_k}$,

$$\frac{\alpha}{\lambda} = 1 - \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I} + \sum_{i=1}^{n} t_i \frac{S_{ik}}{X_k}. \tag{9.55}$$

From (9.55) it can be seen that the divergence of $\frac{\alpha}{\lambda}$ from 1 can be separated into two components: (i) a revenue effect given by $\sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I}$; and (ii) a distortionary effect $\sum_{i=1}^{n} t_i \frac{S_{ik}}{X_k}$. From the negative semi-definiteness of the Slutsky matrix it follows that

$$\sum_{i=1}^{n} t_i \frac{S_{ik}}{X_k} \leq 0. \tag{9.56}$$

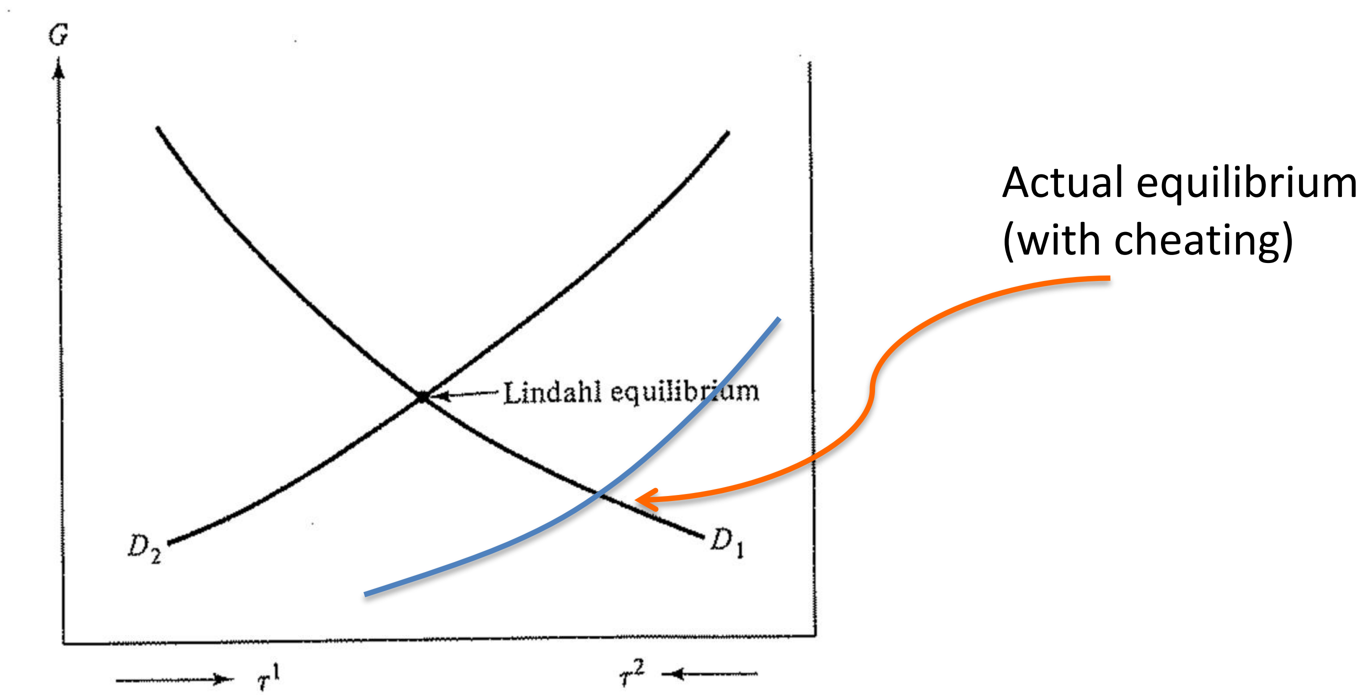
The negativity in (9.56) has the effect of tending to reduce α below λ. This would imply that the true benefit of the public good is less than the $\sum MRS$.

In contrast, the second effect $\sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I}$ cannot be unambiguously signed. If it were positive then α would be less than λ. This would be the case if all taxed goods were normal, but this may not be the case.

Comments on Lindhal equilibrium

- The analysis of the Lindahl equilibrium assumed that households were honest in revealing their reactions to the announcement of cost shares.
- However, there will be a gain to households who attempt to cheat, or manipulate, the allocation mechanism.
- By announcing preferences that do not coincide with their true preferences, it is possible for a household to modify the outcome in their favour provided that others do not do likewise.

Lindhal equilibrium



Due to this problem, attention has focused upon the design of allocation mechanisms that overcome attempted manipulation. The design of some of these mechanisms leads households to reveal their true preferences.

Mechanism design: introduction

- The general form of allocation mechanism can be described as a game in which each household has a strategy set and chooses a strategy from this set in order to maximise their pay-off.
- The aim of the analysis is to determine when a game can be constructed such that the equilibrium strategies lead to the allocation that the policy maker wishes to see implemented.

Introduction

- The game will be one of incomplete information since it is natural to assume that each household has knowledge only of its own payoff function.
- Most attention in the early literature was upon the dominant strategy equilibrium, where each household has a dominant strategy regardless of the choices of others, and the Nash equilibrium in which the chosen strategy must be optimal given that other households play their equilibrium strategy.

Definitions

- Each household is assumed to have additively separable preferences given by

$$U^h(G, t_h) = \nu^h(G) + t_h.$$

- v is the valuation of the project, t is lump sum tax/payment part of the game.
- Decision d (i.e. G) is made by the centre (or policymaker, choosing G and transfers), based on announcements of (or reported) valuations by households, w^h concerning the provision of good G.

Direct revelation mechanism

The chosen project maximises the sum of reported valuations and is therefore optimal given those valuations:

$$d\left(w\right) \;=\; d\left(w^{1}\left(\cdot\right),...,w^{H}\left(\cdot\right)\right) \;\in\; \left\{G^{*}: G^{*}=\arg\max_{\{G^{*}\in\mathcal{G}\}}\sum_{h=1}^{H} w^{h}\left(G\right)\right\}.$$

A DRM with decision function d(w) and associated transfers $\{t^h (w)\}$ is termed strongly individually incentive compatible (s.i.i.c.) if truth-telling is a dominant strategy, that is, iff for all agents telling the truth maximizes pay-off of h)

$$\nu^{h}\in\arg\max_{\{w^{h}\in V^{h}\}}\nu^{h}\left(d\left(w^{h},w^{-h}\right)\right)+t_{h}\left(w^{h},w^{-h}\right),\quad\forall w^{-h},\quad h=1,...,H, \tag{9.64}$$

From the definition of a DRM it follows that such as.i.i.c. DRM: d (w) has the property that (if it does exist): SOCIAL OBJECTIVE FUNCTION as the sum of total evs.

$$\nu^{h}\in\arg\max_{\{w^{h}\in V^{h}\}}\nu^{h}\left(d\left(w^{h},w^{-h}\right)\right)+\sum_{j=1,j\neq h}^{H}w^{j}\left(d\left(w^{h},w^{-h}\right)\right),\quad h=1,...,H. \tag{9.65}$$

Write the tax function as:

$$t_{h}\left(w\right)=\sum_{j=1,j\neq h}^{H}w^{j}\left(d\left(w\right)\right)+r^{h}\left(w\right),\quad h=1,...,H,$$

Hence, (9.64) becomes:

Groves mechanism

- INDIVIDUAL OBJECTIVE FUNCTION:

$$\nu^h \in \arg \max_{\{w^h \in V^h\}} \nu^h \left(d\left(w^h, w^{-h}\right)\right) + \sum_{j=1, j\neq h}^{H} w^j \left(d\left(w^h, w^{-h}\right)\right) + r^h \left(w^h, w^{-h}\right), \quad h = 1, ..., H. \tag{9.67}$$

Social and indvidual o.functions yield the same result if rh is independent of w^h for each individual h.

$$t_h(w) = \sum_{j=1, j\neq h}^{H} w^j (d(w)) + r^h \left(w^{-h}\right), \quad h = 1, ..., H,$$

This is known as Groves mechanism
Theorem: 9.6: a Grove mechanism is s.i.i.c.

- Interpretation: the transfers are such that the only effect the strategy choice of a household can have upon the size of the transfer is via the effect that the decision on the public project, based upon that strategy, has upon other households' welfare. i.e.: There is no direct effect on the transfer.
- This mechanism can be viewed as internalising the external consequences of the strategy, given by the welfare effects on other households of the public decision

Clarke solution as a special case

$$r^h\left(w^{-h}\right) = -\sum_{j=1, j\neq h}^{H} w^j\left(d_h\left(w^{-h}\right)\right), \quad h = 1, ..., H, \tag{9.69}$$

where $d_h\left(w^{-h}\right)$ is the maximiser of $\sum_{j=1,j\neq h}^{H} w^j\left(G\right)$. In this case the transfer is exactly the change in welfare of other households due to influence of household h on the public project decision. This is a special case of the Groves mechanism.

Comments

- No other mechanism does provide the same result;
- Based on strong assumptions (utility separability)
- Parameters restrictions for the transfer system to be balanced
- More general Nash equilibria obtained under strong information requirements.
- Mechanism design is still an open issue