Commodity taxation

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Public economics

(Chapter 5 Myles: section 4,5,6,8 excluded)

Introduction

- The analysis restricts the set of feasible policy instruments available to the government to commodity taxes.
- The use of optimal lump-sum taxes is assumed to be prevented: the relevant characteristics are preferences and endowments but these are private information and will not be truthfully revealed under the optimal tax system.

Cont'd

- Employment of commodity taxes requires only that the government is able to observe trades in commodities.
- Although it may be possible for the government to levy a uniform lump-sum tax, and in a one-household economy such a tax would also be optimal, it is assumed for simplicity that such taxes cannot be employed.
- In an economy where the households are not identical, their introduction does not significantly modify the conclusions.

Cont'd

- Standard methodology in optimal commodity tax theory: only linear taxes, either additive so that the post-tax price of good i is given by pi + ti or multiplicative with post-tax price .
- A social welfare function is then maximised by choice of the tax rates and the first-order conditions for this maximisation are manipulated to provide a qualitative description of the optimal tax system.
- Difficult to get explicit results. Numerical simulation are usual.
- Normalisation rules employed, so the actual values of tax rates can be argued to have little meaning.

Cont'd

- It is the real effect of the tax system upon the equilibrium quantities of each good that is relevant.
- Mirrlees (1976): the index of discouragement is introduced to measure the effect of taxes.
- Secondly, the economies of this chapter assume that households trade only with firms (otherwise nonlinearities in individual's budget constraints).

Ramsey rule

- The model:
- n consumption goods and a single form of labour, which is the only input.
- Each industry produces a single output with CRS
- Single household (or identical consumers), with preferences represented by indirect utility function.

Assumptions

- For each good i there is a coefficient c^i describing the labour input necessary to produce 1 unit of that good: $y^i/L^i=c^i$
- Perfect competition implies that profit maximization:
- $p^i=c^i w$, i=1,…n
- Where p^i is the pre-tax price.
- Labour is chosen as the numeraire, w is fixed.
- Hence, we have a set of fixed pre-tax (or producer prices) for the consumption goods.

Assumptions

- Post-tax or consumer prices are:
- $q^i=p^i+t^i$, $i=1,\ldots n$

- Taxes on goods are levied so as to satisfy a total revenue constraint R:
- $R=\Sigma_i t_i x_i$
- Taxes are used by the State to buy certain goods (labour) that are not traded in the market (e.g. defence system).

Preferences

- Indirect utility functions:
- $U=V(q_1, q_2, \ldots q_n, w, I)$
- Where I is Lump-sum Income. w comes from labour supply for production of goods and for the State. Perfect competition implies zero profits and so no profit income and I=0.

Derivation

- The optimal tax problem is:

$$\max_{\{t_1,...,t_n\}} V(q_1, ..., q_n, w, I) \quad \text{subject to} \quad R = \sum_{i=1}^{n} t_i x_i. \tag{4.5}$$

The Lagrangean corresponding to (4.5) is given by

$$L = V(q_1, ..., q_n, w, I) + \lambda \left[\sum_{i=1}^{n} t_i x_i - R \right]. \tag{4.6}$$

From (4.6), the first-order necessary condition for the choice of tax rate on good k is

$$\frac{\partial L}{\partial t_k} \equiv \frac{\partial V}{\partial q_k} + \lambda \left[x_k + \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k} \right] = 0, \tag{4.7}$$

where the identities

$$\frac{\partial V}{\partial q_k} \equiv \frac{\partial V}{\partial t_k}, \frac{\partial x_i}{\partial q_k} \equiv \frac{\partial x_i}{\partial t_k}, \tag{4.8}$$

have been used. Equation (4.7) can be rearranged to give

$$\frac{\partial V}{\partial q_k} = -\lambda \left[x_k + \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k} \right], \tag{4.9}$$

- For all goods i.
- Interpretation: for all goods the welfare cost of raising a tax on good k should be proportional to the marginal revenue brought about by the tax rise.
- From Roy's identity: $\frac{\partial V}{\partial q_k} = -\frac{\partial V}{\partial I} x_k = -\alpha x_k,$

$$\alpha x_k = \lambda \left[x_k + \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k} \right]. \tag{4.11}$$

After rearrangement (4.11) becomes

$$\sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k} = - \left[\frac{\lambda - \alpha}{\lambda} \right] x_k. \tag{4.12}$$

The next step in the derivation is to employ the Slutsky equation to note that

$$\frac{\partial x_i}{\partial q_k} = S_{ik} - x_k \frac{\partial x_i}{\partial I}. \tag{4.13}$$

The right-hand side of (4.15) is now simplified by extracting the common factor x_k which yields

$$\sum_{i=1}^{n} t_i S_{ik} = -\left[1 - \frac{\alpha}{\lambda} - \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial I}\right] x_k. \tag{4.16}$$

- The symmetry of the Slutsky substitution matrix implies that $S_{ki} = S_{ik}$. This symmetry can be used to rearrange (4.16) to give the expression

$$\sum_{i=1}^{n} t_i S_{ki} = -\theta x_k, \theta = \left[1 - \frac{\alpha}{\lambda} - \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial I}\right]. \tag{4.17}$$

- For all goods k
- This is the Ramsey rule describing a set of optimal commodity taxes. Note that theta is independent of the particular good chosen. Finally, multiplying by t_k and summing over k

$$\sum_{k=1}^{n} \sum_{i=1}^{n} t_i t_k S_{ki} = -\theta R.$$

- Sign

Interpretation

$$S_{ki} = \frac{\partial \chi_k}{\partial q_i}, \tag{4.19}$$

where χ_k is the Hicksian or compensated demand for good k. Consequently, starting from a position with no taxes, and noting that t_i is then the change in the tax rate on good i,

$$t_i S_{ki} = t_i \frac{\partial \chi_k}{\partial q_i}, \tag{4.20}$$

is a first-order approximation of the change in compensated demand for good k due to the introduction of the tax t_i, but with the property that the derivative is evaluated at the final set of prices and at post-tax utility level. If the taxes are small, this should be a good approximation. Extending this argument to the entire set of taxes, it follows that

$$\sum_{i=1}^{n} t_i S_{ki}, \tag{4.21}$$

is an approximation to the total change in compensated demand for good k due to the introduction of the tax system from an initial no-tax position.

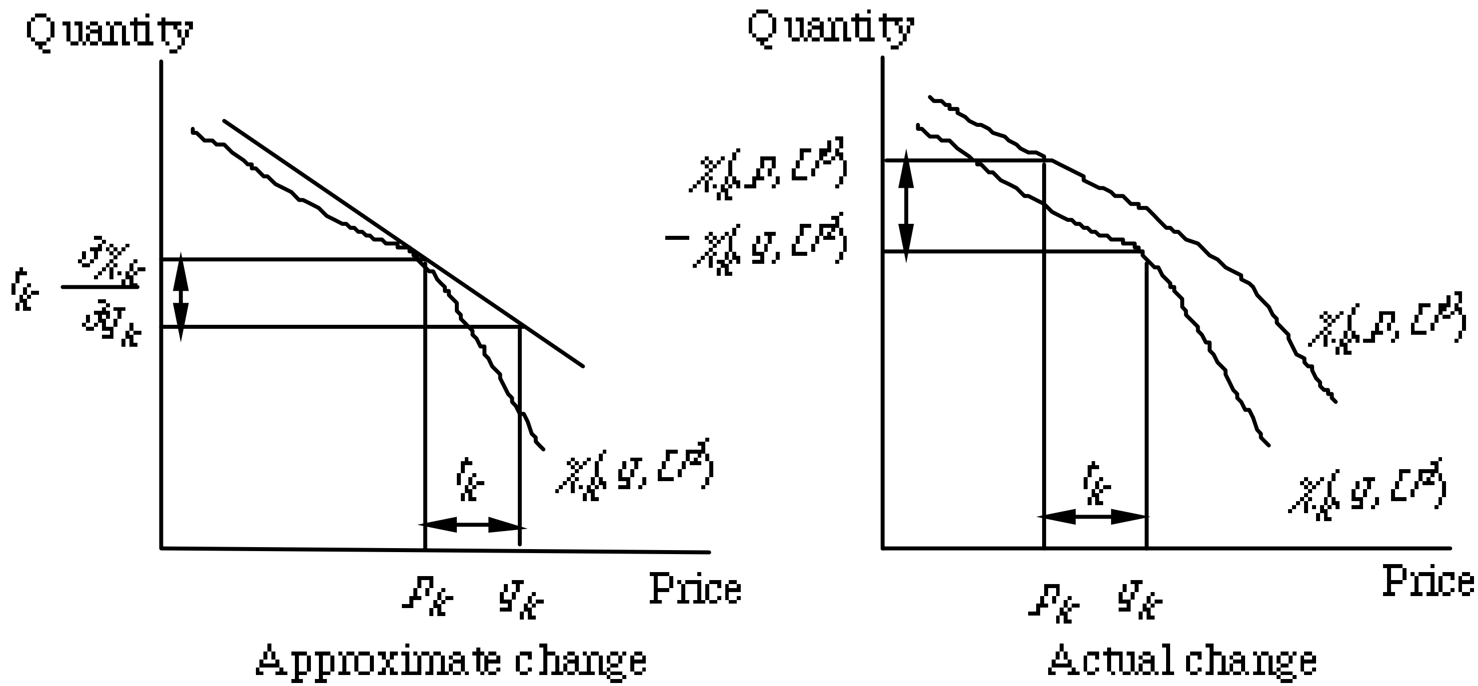

Figure 4.1: Interpretation of the Ramsey Rule

$$\frac{\sum_{i=1}^{n} t_i S_{ki}}{x_k} = -\theta, k = 1, ..., n,$$

- Ramsey rule can be interpreted as follows: the optimal tax system should be such that the compensated demand for each good is reduced in the same proportion relative to the pre-tax position.

Concluding

- By calling

$$d_k = \frac{\sum_{i=1}^{n} t_i S_{ki}}{x_k},$$

Where d_k ,the proportional reduction in demand, is Mirrlees' (1976) index of discouragement. The Ramsey rule states that the tax system is optimal when the index of discouragement is equal for all goods.

Implications (1)

- Since the proportional reduction in compensated demand must be the same for all goods it should follow that goods whose demand is unresponsive to price changes will bear higher taxes.
- Although correct, in general, this statement can only be truly justified when all cross-price effects are accounted for.

Implications (2)

- Goods unresponsive to price changes are typically necessities (housing, food).
- Consequently, the implementation of a tax system based on the Ramsey rule would lead to taxes that would bear most heavily on necessities, with the lowest tax rates on luxuries.
- This interpretation has been demonstrated more formally by Deaton (1981) under the assumption of weak separability of preferences.
- Highly inequitable solution corrected under heterogeneity

Implications (3)

- The equilibrium determined by the set of optimal taxes is second-best compared to the outcome that would arise if the tax revenue had been collected via a lump-sum tax.
- In fact, commodity taxes lead to substitution effects which distort the household's optimal choices and lead to efficiency losses.
- Although unavoidable when commodity taxes are employed, these losses are minimised by the optimal set of taxes that satisfy the Ramsey rule.

Inverse elasticity rule (1)

- See Baumol and Bradford (1970).
- It is derived by assuming that there are no cross-price effects between the taxed goods so that the demand for each good is dependent only upon its own price and the wage rate.
- the general equilibrium model turns into one of partial equilibrium as it removes all the interactions in demand and, as shown by Atkinson and Stiglitz (1980), the inverse elasticities rule can be derived from minimising the excess burden of taxation in a partial equilibrium framework.

Inverse elasticity rule (2)

$$\alpha x_k = \lambda \left[x_k + \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k} \right]. \qquad (4.25)$$

The assumption of independent demands implies that

$$\frac{\partial x_i}{\partial q_k} = 0 \text{ for } i \neq k. \qquad (4.26)$$

Employing (4.26), equation (4.25) reduces to

$$\alpha x_k = \lambda \left[x_k + t_k \frac{\partial x_k}{\partial q_k} \right]. \qquad (4.27)$$

Rearranging (4.27) and dividing by q_k, where by assumption $q_k = p_k + t_k$, gives

$$\frac{t_k}{p_k + t_k} = \left[\frac{\alpha - \lambda}{\lambda} \right] \left[\frac{x_k}{q_k} \frac{\partial q_k}{\partial x_k} \right]. \qquad (4.28)$$

Inverse elasticity rule (3)

As

$$\frac{x_k}{q_k}\frac{\partial q_k}{\partial x_k} = \frac{1}{\varepsilon_k^d}, \tag{4.29}$$

where ε_k^d is the price elasticity of demand for good k, (4.28) can be written

$$\frac{t_k}{p_k + t_k} = \left[\frac{\alpha - \lambda}{\lambda}\right]\frac{1}{\varepsilon_k^d}. \tag{4.30}$$

- The inverse elasticities rule states that the proportional rates of tax should be inversely related to the price elasticity of demand of the good on which they are levied.
- Necessities, which by definition have low elasticities of demand, should be highly taxed.
- Strong assumptions and absence of heterogeneity.

Production efficiency

- Production efficiency occurs when an economy is maximising the output attainable from its given set of resources.
- In the special case in which each firm employs some of all of the available inputs, a necessary condition for production efficiency is that the marginal rate of substitution (MRS) between any two inputs is the same for all firms

Production efficiency

- Such a position of equality is attained, in the absence of taxation, by the profit maximisation of firms in competitive markets.
- Each firm sets the marginal rate of substitution equal to the ratio of factor prices and, since factor prices are the same for all firms, this induces the necessary equality in the MRSs
- The same is true when there is taxation provided all firms face the same post-tax prices for inputs, that is, inputs taxes are not differentiated between firms.

Production efficiency: Lemma

- Diamond & Mirrlees (1971): Production Efficiency lemma.
- In a competitive economy, the equilibrium with optimal commodity taxation should be on the frontier of the aggregate production set.
- This can only be achieved if private and public producer face the same shadow prices and if input taxes are not differentiated between firms.

Lemma

- In addition, since the competitive assumption implies that any set of chosen post-tax prices can be sustained by the use of taxes on final goods alone, the latter statement also carries the implication that intermediate goods should not be taxed .
- This result was seen as surprising because in contrast to the predictions of the Lipsey-Lancaster (1956) Second-Best theory that was being widely applied.
- Second-Best theory, which typically suggests that one distortion should be offset by others, would imply that the distortion induced by the commodity taxes should be matched by a similar distortion in input prices.

Proposition

Lemma 18 *(Diamond and Mirrlees) Assume that social welfare is strictly increasing in the utility level of all households. If either*

(i) for some i, $x_i^h \leq 0$ *for all* h *and* $x_i^{\widehat{h}} < 0$ *for some* $\widehat{h}$*;*

or

(ii) for some i*, with* $q_i > 0$, $x_i^h \geq 0$ *for all* h *and* $x_i^{\widehat{h}} > 0$ *for some* $\widehat{h}$*;*

then if an optimum exists, the optimum has production on the frontier of the production possibility set.

- Proof. Assume the optimum is interior to the production set. In case (i), increasing q_i would not reduce the welfare of any household and would strictly raise that of h^.
- Such a change is feasible since the optimum is assumed interior and the aggregate demand function is continuous. The change would raise social welfare, thus contradicting the assertion that the initial point was optimal. The same argument can be applied in case (ii) for a reduction in q_i.

Comments

- The Diamond-Mirrlees lemma therefore provides an argument for the non-taxation of intermediate goods and the non-differentiation of input taxes between firms.
- It has been extended from its original constant returns to scale setting. However, except for some special cases, imperfect competition invalidates the lemma and taxes on intermediate goods will raise welfare. --