Public Economics

Recent developments on the theory of dynamic optimal taxation

By Luca Spataro

Introduction

What is it about?

The problem is the choice of optimal taxation in the absence of lump sum taxes to finance an exogenous stream of public expenditure (Ramsey (1928) problem)

Chamley (1986) and Judd (1985) came out with a relatively stricking result:

- “taxing capital income in the long run is a bad idea”.
- “in the short run, it depends on preferences”
- This result is quite robust!

Background

Where does this result stem from?

We had to wait up to 1999: (Judd, JPubE 1999) and others in order to correctly grasp the intuition behind the result:

Equivalence between tax on capital income and tax on future consumption;

which implies that future consumption should be taxed only if the elasticity of consumption (H_c) varies over time.

Background

- Such a result remained unchallenged (and, if possible enhanced) for a long time, up to 2002, when some exceptions were unveiled by the literature (Erosa and Gervais, JET 2002 for OLG models);
- then other authors came out with other cases invalidating the zero tax result (for example, De Bonis-Spataro, MD 2005 and EM2008 and OEP09, Reis 2011, JET);

Outline

First part:

- The zero tax result in the ILRA (Infinitely Lived Representative Agent), standard model;
- Relaxing some hypotheses: the result still holds

Second part:

- Some exceptions both in ILRA and in OLG models;
- In fact, we will show that:
- **In Infinitely Lived Agent Model: zero tax is the rule;**
- **Overlapping Generations Economy: zero tax is rather an exception**.

Methodological toolbox

1) The framework combines two traditions in economics:
2) the public finance (PF) tradition;
3) The general equilibrium (GE) tradition;

The PF tradition starts from Ramsey (1928): choosing the optimal structure of taxes in an economy with RAs when only distortionary taxes are available;

The GE tradition is concerned with growth models arising from consumers' optimal choices of consumption and investment. It dates back to Cass (1965), Koopmans (1965), Kydland and Prescott (1982), Lucas and Stockey (1983)

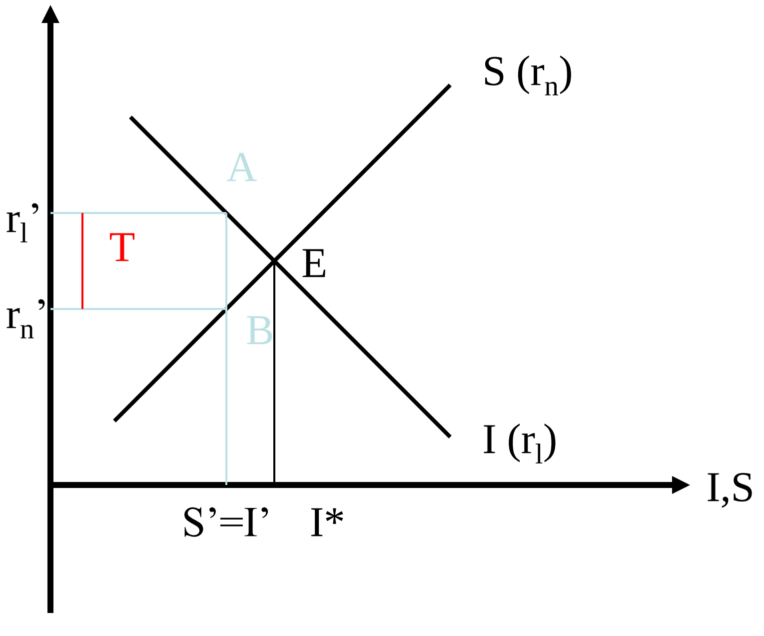
Hence:

a) General equilibrium framework
b) Dynamic context
c) Primal approach to the Ramsey problem

The fiscal wedge and deadweight loss

- The tax introduces a wedge between the gross interest yield paid by those who demand for capital (r_l), and the net of tax yield (r_n), obtained by net suppliers of capital; Investments overall decrease (I).
- The distortion of the intertemporal choice is thought to be particularly harmful for its effects on productive activities of the economy.

$$\frac{EB}{G} = \frac{1}{2}\frac{Tdq}{Tq} = \frac{1}{2}\frac{dq}{q}$$



The Ramsey's rule in the static, traditional framework

In order to minimize the excess burden of (distortionary) taxes, the indirect taxes must be implemented in such a way that they cause the same proportional reduction of demand for each good;

$$\frac{EB}{G}=\frac{1}{2}\frac{Tdq}{Tq}=\frac{1}{2}\frac{dq}{q} \quad \Rightarrow \quad \frac{EB_e}{G_e}=\frac{EB_r}{G_r} \quad \Rightarrow \quad \frac{dq_e}{q_e}=\frac{dq_r}{q_r}$$

From this law, the Ramsey rule holds: if two goods are "not connected" in consumption, the optimal indirect tax rates should be inversely proportional to the price elasticity of the demand of goods

$\varepsilon=\frac{dq}{dp}\frac{p}{q} \quad \Rightarrow \quad \frac{dq}{q}=\frac{\varepsilon(dp)}{p}$; let $p=1$ Knowing that dp=T $\Longrightarrow$

$\Longrightarrow \frac{dq}{q}=\varepsilon T$ thus $\varepsilon_e T_e=\varepsilon_r T_r$ $\quad \frac{\varepsilon_e}{\varepsilon_r}>1 \Rightarrow T_r>T_e$

The ILRA Model (based on Atkeson, Chari, Kehoe (1999). See e-learning to download)

Hypotheses

- Closed, competitive, production economy populated by a large number of identical, infinitely lived consumers and firms;
- In the economy there are two goods: consumption-capital and labour;
- Technology: CRS with Inada conditions being satisfied:
- The government levies distortionary taxes so as to finance an exogenous stream of public good;

Private agents

•**Private Agents solve the following problem**

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \qquad U_c > 0 \quad U_l < 0 \quad U_{c,c} < 0 \quad U_{l,l} < 0$$

s.t.

$$a_{t+1} = \left(1 + \tilde{r}_t\right) a_t + \tilde{w}_t l_t - c_t \qquad \text{with} \quad \tilde{r}_t = r_t(1 - \tau_t^k) \quad \tilde{w}_t = w_t(1 - \tau_t^l)$$

+ No-Ponzi on final wealth + initial condition on wealth (a_0 given)

Focs'

$$\frac{U_l}{U_c} = -\tilde{w}_t$$

$$\frac{U_c}{U_{c+1}} = \frac{p_t \beta}{p_{t+1}} = \beta(1 + \tilde{r}_{t+1})$$

- Firms

- Perfect competition, CRS technology
- Profit maximization condition yields

$$\max \; F(k_t, l_t) - r_t k_t - w_t l_t - \delta k_t$$

FOCs

$$\frac{\partial \Pi(k_t, l_t)}{\partial k_t} = 0 \Rightarrow F_k(k_t, l_t) = r_t + \delta$$

δ is the depriciation rate of capital

$$\frac{\partial \Pi(k_t, l_t)}{\partial l_t} = 0 \Rightarrow F_l(k_t, l_t) = w_t$$

•GOVERNMENT

The government finances an exogenous amount of public expenditure G by levying taxes on labor and capital income and by issuing debt;

-no lump sum taxation;

- aggregate debt dynamics:

$$b_{t+1} = (1 + r_t)b_t + g_t - \tau_t^k r_t a_t - \tau_t^l w_t l_t$$

•CAPITAL MARKET CLEARING CONDITION

•$a_t = b_t + k_t$

•The time inconsistency problem

•we assume that the State has a commitment technology in order to tie its hands against deviating from the annunciated ex-ante optimal policy

Competitive equilibrium

A competitive equilibrium is a sequence of

- Policies, $\pi = \{\tau^k, \tau^l, b_{t+1}\}_0^\infty$
- Allocations; $\{c_t, l_{t,} k_{t+1}\}_0^\infty$
- Prices; $\{w_t, r_t\}_0^\infty$

Such that

- Individuals and firms solve their maximization problems, markets clear and resource constraints and public budget constraints are satisfied

Ramsey equilibrium

Primal approach: government maximizes a social welfare function by choosing an allocation among those that:

(a) can be decentralized as a competitive equilibrium

(b) are feasible

(c) compatible with the government budget constraint

The primal approach

a) implementability constraint: dynasty budget constraint with prices substituted out from FOCs (individuals are on their offer curve)

$$\sum_{t=0}^{\infty} \beta^t \left(U_c c_t + U_t l_t \right) = \left(1 + \tilde{r}_0 \right) a_0 U_{c_0}$$

b) feasibility constraint: resource constraint

$$c_t + k_{t+1} - k_t (1 - \delta) + g_t = F(k_t, l_t)$$

c) Debt equation: omitted by Walras' law

Proposition 1: An allocation is a competitive equilibrium if and only if it satisfies implementability and feasibility constraints

Sketch of the proof

Preliminarily let us get the lifetime budget constraint of an individual (see notes on the elearning website)…

Solution of the Ramsey problem

Max social welfare function, under implementability and feasibility

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

s.t.

$$\sum_{t=0}^{\infty} \beta^t \left(U_c c_t + U_l l_t\right) = \left(1 + \tilde{r}_0\right) a_0 U_{c_0}$$

+ feasibility for every t>0 + transv'ty

Let us denote with W pseudo-welfare function:

$$W_t = U(c_t, l_t) + \lambda(U_c c_t + U_l l_t)$$

Where lambda is the Lagrange multiplier of the impl'ty constr't and is interpreted as the deadweight loss of distortionary taxation. Hence the problem can be stated as:

$$Lagr = \sum_{t=0}^{\infty} \beta^t W_t - \lambda\left(1 + \tilde{r}_0\right) a_0 U_{c_0} - \sum_{t=0}^{\infty} \gamma_t \left[c_t + k_{t+1} - k_t(1-\delta) + g_t - F(k_t, l_t)\right]$$

With γ_t >0, the Lagrange multiplier of the feasibility constraint (shadow price of capital).

Solution of the Ramsey problem

Hence, by maximizing with respect to l(t), c(t), k(t+1) and c(t+1) (see notes) we get

$$-\frac{W_l}{W_c} = -\frac{U_l\left[1+\lambda\left(1+H_l\right)\right]}{U_c\left[1+\lambda\left(1+H_c\right)\right]} = F_{l_t}$$

$$\frac{W_{c_t}}{W_{c^{+1}}} = \frac{U_c\left[1+\lambda\left(1+H_c\right)\right]}{U_{c^{+1}}\left[1+\lambda\left(1+H_{c^{+1}}\right)\right]} = \beta\left(1+F_{k_{t+1}}-\delta\right)$$

$$H_l \equiv \frac{U_{c,l}c_t + U_{l,l}l_t}{U_l}$$

$$H_c \equiv \frac{U_{c,c}c_t + U_{l,c}l_t}{U_c}$$

Where H is the general equilbrium elasticity of labour or consumption

Solution of the Ramsey problem

From the equations above and exploiting the individuals' and firms' Focs, we get the implicit expression of the tax rate on wages:

$$\frac{\tilde{w}}{w} = \frac{[1+\lambda(1+H_c)]}{[1+\lambda(1+H_l)]} \Rightarrow \tau_t^l = \lambda \frac{H_l - H_c}{[1+\lambda(1+H_l)]}$$

and on the interest rate:

$$\frac{1+\tilde{r}^{+1}}{1+r^{+1}} = \frac{1+(1-\tau_k^{+1})r^{+1}}{1+r^{+1}} = \frac{[1+\lambda(1+H_c^{+1})]}{[1+\lambda(1+H_c)]}$$

Results

PROPOSITION 2

If the economy converges towards a steady state, then in the long run the capital income tax should be zero

$$\frac{1+\tilde{r}^{+1}}{1+r^{+1}} = \frac{1+(1-\tau_k^{+1})r^{+1}}{1+r^{+1}} = \frac{[1+\lambda(1+H_c^{+1})]}{[1+\lambda(1+H_c)]}$$

PROPOSITION 3a

Along the transition path, the capital income tax is zero if the utility function is separable between capital and labour and homothetic in consumption, for t>1. For example:

$$U(c,l) = c^{1-\sigma} \big/ (1-\sigma) + V(l) \quad \text{with} \quad \sigma = -\frac{1}{H_c}$$

What about the first period?

$$W_0 - \lambda\left(1 + \tilde{r}_0\right) a_0 U_{c_0}$$

So, First order conditions for the policymaker are, in general, different from the others. Hence there is room for hitting capital income.

How much?

Constraints on the capital income tax along the transition

PROPOSITION 3b

For utility functions of the form seen above, in case there is an upper bound on the tax rate, the latter is binding for a finite number of periods. After that, the tax takes an intermediate value for one period and is zero thereafter.

Sketch of the Proof (see notes)