Public Economics

Recent developments on the theory of dynamic optimal income taxation: 2

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Summary of the results

• **long run**: the constant elasticity of consumption \((Hc^{+1}=Hc)\) implies **no taxation** of capital income at the steady state

• **short run**: depends on utility function (e.g.: no taxation if (weakly separable in consumption and leisure and homothetic in consumption): otherwise Corlett-Hauge rule (1953))
Taxing capital income is a bad idea

In the long run, only labor income should be taxed in order to raise taxes for financing public expenditures.

Intuition: it is the equivalent of the Ramsey’s rule in a dynamic context. In fact, there is an equivalence between tax on capital income and tax on future consumption. This implies that future consumption should be taxed only if the elasticity of consumption (Hc) varies over time (Judd, JPubE 1999).

More precisely: a (constant) positive capital income tax is equivalent to an ever increasing tax on consumption.

\[
\frac{\Delta C_1}{\Delta C_0} = -[1+\tilde{r}] \quad \tilde{r} = r(1-\tau_k)
\]

\[
\Rightarrow \frac{\Delta C_2}{\Delta C_0} = \frac{\Delta C_2}{\Delta C_1} \frac{\Delta C_1}{\Delta C_0} = -[1+r(1-\tau_k)]^2 \Rightarrow \frac{\Delta C_n}{\Delta C_0} = -[1+r(1-\tau_k)]^n
\]

According to the Ramsey’s rule, this is optimal only if the elasticity of consumption is decreasing through time. But, given that in the ILRA model, at the s.s., consumption is constant, it follows that consumption cannot be taxed in the long run. Possibly, there is room to tax capital income in the short run.
Is this result robust? Relaxing some hypotheses

- Heterogeneous agents (either with different preferences or savers and consumers)
- Endogenous growth
- Open economy
First part

• Relaxing assumptions: heterogeneous agents

• Workers and savers
Second part

Violations of the zero tax result

• Market imperfections (Guo-Lansing 1999)
• Restrictions on taxes (Correia 1996)
• ILRA models: difference between government and individuals’ discount rates (De Bonis-Spataro 2005, MD)
• Overlapping generations:
  - LC motive (Erosa-Gervais (2002) JET;
• A) Difference between government and private agents’ intertemporal discount rates
Solution of the Ramsey problem

Max social welfare function, under implementability and feasibility

\[
\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)
\]

s.t.

\[
\sum_{t=0}^{\infty} \beta^t (U_c c_t + U_l l_t) = (1 + \tilde{r}_0) a_0 U_{c_0} + \text{feasibility for every } t > 0
\]

Let us denote with \( W \) a pseudo-welfare function of the form:

\[
W_t = U(c_t, l_t) + \lambda_t (U_c c_t + U_l l_t)
\]

Where

\[
\lambda_t = \lambda \left( \frac{\beta}{\beta_g} \right)^t
\]
Hence, the solution to the Ramsey problem takes the following form:

\[
\frac{1 + \tilde{r}^{+1}}{1 + r^{+1}} = \frac{\beta_g}{\beta} \left[ 1 + \lambda_t^{+1} (1 + H_c^{+1}) \right]
\]

If, for the sake of simplicity, we assume that \( H_c = H_c(1) \):

1) \( \beta_g = \beta \Rightarrow \lambda_t^{+1} = \lambda_t \Rightarrow \tau^k = 0 \)

What do you expect in the other cases?
2) $\beta_g > \beta$

$t \to \infty \Rightarrow \lambda_t \to 0 \Rightarrow \frac{\beta_g}{\beta} \frac{1 + \lambda_t^{+1} (1 + H_c)}{1 + \lambda (1 + H_c)} = \frac{\beta_g}{\beta}$

$\Rightarrow \frac{1 + \tilde{r}^{+1}}{1 + r^{+1}} = \frac{\beta_g}{\beta} \Rightarrow \tau^k < 0$

3) $\beta_g < \beta$

$t \to \infty \Rightarrow \lambda_t \to \infty$

$\Rightarrow \frac{1 + \tilde{r}^{+1}}{1 + r^{+1}} = \frac{\lambda_t^{+1}}{\lambda_t} \frac{\beta_g}{\beta} \frac{1/ \lambda_t^{+1} + (1 + H_c)}{1/ \lambda_t + (1 + H_c)} = 1 \Rightarrow \tau^k = 0$

Intuition: Recall that lambda is the value of the distortion brought about by the distortionary taxation

In the case 2, the gov’n’t is more **patient** than individuals, eager to favor future consumption=>it incentivates savings via negative taxes on capital income. This can be interpreted in terms of Pigouvian taxation (social MRS is different from the private MRS).

In case 3, the gov’n’t is more **impatient** than individuals ( weighs present consumption more than individuals do) so it would like to hit future consumption by levying positive taxes on future capital income. However, it is not optimal to do that: the (present) value of the distortion goes to infinite, so that the gain in the welfare obtained by levying taxes is overwhelmed by the loss of welfare due to the distortion. All in all, it is better not to tax capital income.
B) Overlapping generation Economies: the role of
   i) life-cycle
   ii) Disconnection of the economy (e.g. immigration)
Overlapping Generations Economy

Hypotheses: in each period new individuals (generations) enter the economy (e.g. migrants), while others die (with certainty or not, it does not matter);

in each period, the cross section of the economy is formed by OVERLAPPING GENERATIONS (OLG): individuals are identical but have different ages.

What is crucial here?

• The presence of life-cycle behavior
• (Some) Individuals (or generations) are disconnected from the other (already existing or future) individuals or generations (Weil 1989 JPuEc).
The model (partial altruism)

Demographics

\( \alpha: \) immigration rate

\( n: \) dynasty growth rate

\( \gamma \equiv (1+\alpha)(1+n)-1: \) population growth rate

\( N_t = N_0 (1+\gamma)^t: \) size of population at time \( t \)

\( P_{s,t} = \alpha N_s (1+n)^{t-s}, s \leq t: \) size of each dynasty at time \( t \) (started by the entry of the founder at time \( s \))

\( N_0: \) size of population at time 0

The dynasty size decreases with time because of the entry of new immigrants \( \nu \) \( (t+1)/\nu (t) = 1/(1+\alpha) \)
Results

1) In line with the traditional analysis, scope for a differential treatment of consumption in different life periods (capital income tax) and of own future and descendants’ consumption (inheritance tax) arises if the general equilibrium elasticity of consumption varies between life periods and between generations, respectively.
Ramsey problem

- The policymaker maximizes a utilitarian swf which is a weighted sum of the dynasties utilities, s.t. the above presented constraints hold
- Define the auxiliary function (dynastic U plus dynastic implementability:

\[
W_s = \sum_{t=s}^{\infty} \left( \frac{1+n}{1+\beta} \right)^{t-s} \left[ \mu_{s,t} U (c_{s,t}, l_{s,t}) + \lambda_s (U_{c_s,t}c_{s,t} + U_{l_s,t}l_{s,t}) \right]
\]

where \( \mu_{s,t} \) = weight that the government attaches to dynasty \( s \)

**Remark:** \( \mu_{s,t} \) is allowed to vary with time
Ramsey problem

The policymaker’s problem is the following:

\[ \max_{\{c_{s,t}, l_{s,t}, k_t\}_0} \sum_{s=0}^{t} W_s \]

subject to:

\[ y_t = c_t + k_t - \frac{k_{t-1}}{(1 + \gamma)} + g_t, \quad \forall t \]

\[ \lim_{t \to \infty} \frac{k_t}{\prod_{i=1}^{t} (1 + f_{k_i})} = 0, \quad k_{-1} = \bar{k}. \]
Solution

and, by exploiting the FOC’s from the dynasty and firm maximization problems and by reckoning that $\frac{\nu^t}{\nu^{t+1}} = (1 + \alpha)$, we get:

\[
\frac{1 + r_{t+1} \left(1 - \tau_k^{t+1}\right)}{1 + r_{t+1}} = \frac{\mu^{t+1} + \lambda \left(1 + H_c^{t+1}\right)}{\mu^t + \lambda \left(1 + H_c^t\right)}
\]

which provides the implicit expression for $\tau_k$. The capital income tax is zero only if the RHS of the above expression is equal to 1

Remark: two forces determining $\tau_k$:
1) the dynamics of $H_c$
2) the dynamics of the social intergenerational weight $\mu$. 
What is new? (Erosa and Gervais, JET 2002)

- the elasticity of consumption $H_c$ is not necessarily constant over life, which implies taxation of capital income even at the s.s.

$$\frac{1 + r_{t+1}(1 - \tau_{t+1}^k)}{1 + r_{t+1}} = \frac{\mu + \lambda (1 + H_{t+1}^t)}{\mu + \lambda (1 + H_t^t)}$$

where $\mu$ is the government intergenerational discount rate (usually equal to the cohort’s demographic weight): in E&G 2002 paper $\mu$ is supposed to be constant

**Remark:** capital income taxes are age conditioned
PROPOSITION

The optimal capital income tax for individuals aged $j+1$ is different from zero iff

$$H_{j+1}^c \neq H_j^c$$

If individuals were not disconnected (i.e. full altruism), we would end up with an ILRA (or ILRGenerations) model and the zero tax result would apply.
What is new? (De Bonis Spataro 2008 EM)

An independent role can be played by the weight attached to the individual utility function by the government within the swf: if these weights correspond to the actual demographic weights of existing cohorts, the disconnection (caused by migration or other devices) is the reason for a nonzero tax.
- Recall that dynasty size decreases with time because of the entry of new immigrants $\nu (t+1)/\nu (t)=1/(1+\alpha)$
- If $\mu$ depends on the dynasty relative size, e.g. $\mu=\nu$, then $\mu$ varies with time

$$\frac{1 + \tilde{r}^{t+1}}{1 + r^{t+1}} = \frac{\nu^{t+1} + \lambda \left(1 + H_c^{t+1}\right)}{\nu^t + \lambda \left(1 + H_c^t\right)}$$
Discussion

$T_k > 0$ even if $H_c = H_c^{+1}$ (and thus even in the absence of life-cycle behavior)

**Intuition:**

- the "disconnection" of the economy, properly accounted for by the policymaker, is an independent source of taxation
- the government discriminates future consumption in favor of present one, since it cares less about the future utility of the dynasty than the dynasty does
Discussion

- the presence of a social weight declining over time corresponds to a government time discounting that differs from the private one, so that government intervention can be interpreted in terms of Pigouvian correction:

\[
\max_{\{c_{s,t}, l_{s,t}, k_t\}} \sum_{s=0}^{t} \mu_s \hat{W}_s
\]

where \( \mu_s = \alpha (1 + \alpha)^s \) and

\[
\hat{W}_s = \sum_{t=s}^{\infty} \left( \frac{1 + n}{1 + \beta} \right)^{t-s} \left[ (1 + \alpha)^{-t} U_{s,t} + \frac{(1 + \alpha)^{-s}}{\alpha} \lambda_s (U_{c_{s,t}, c_{s,t}} + U_{l_{s,t}, l_{s,t}}) \right].
\]
Discussion

Note that in all cases the rationale for taxation derives from Pigouvian arguments. In fact, the results would apply even if lump sum taxes were available, i.e. for $\lambda = 0$. In fact, allowing the social weight to vary with time and age turns out to be equivalent to assuming a constant intergenerational discount rate and a social intertemporal discount rate that differs from the individual one. Thus, varying dynasty’s weights lead the government to correct private accumulation of capital.
Conclusions

• The Chamley-Judd rule does not apply if OLG models, i.e. if there is LC behavior and the economy is disconnected and the policymaker consequently attaches to each dynasty a weight that corresponds to its actual share within the population: the optimal rule implies positive taxation of capital income proportionally to the rate of new migrants arrivals.

The positive taxation of capital income is optimal since it represents a Pigouvian correction.

• Differently from the existing OLG-LC models (e.g. Erosa and Gervais, JET 2002) in DB-S2008, once the role of demographic dynamics is properly taken into account in the social maximization problem, the mere existence of the OLG mechanism (i.e. limited, intradynastic altruism) is sufficient to deliver a non zero capital income tax result, even in the absence of life cycle behavior (so that finite horizons are not necessary).
References

- Reis (2014)