Employment protection with fixed wages

Firms operate in two period. In the first one $Y = \log N$ while in the second

it is $Y = \theta \log N$ where θ is a productivity parameter

with probability *p* it is $\theta_L < 1$ and with *1-p* it is $\theta_H > 1$. Wage is fixed at *W,* price are normalized to 1 and employment is *N.* Firing costs are equal to *z*

We then have the following expected profits $E(\pi)$:

$$E(\pi) = Y_1 - WN_1 + E[Y_2 - WN_2 - z \cdot \max \quad (N_1 - N_2, 0)] \qquad (1)$$

And specifying the production functions:

$$E(\pi) = \log N_1 - WN_1 + p\theta_L \log N_2 + (1-p)\theta_H \log N_2 - WN_2 - z \cdot \max \quad (N_1 - N_2, 0) \qquad (2)$$

We solve it with backward induction

Second period decisions:

If we have θ_H (with probability *1-p*), the firms necessarily want to increase N_1 and $N_1 - N_2 < 0$. Then second period profits are:

$$\pi_2 = \theta_H \log N_2 - WN_2 - z \cdot \max \quad (N_1 - N_2, 0) = \theta_H \log N_2 - WN_2 \qquad (3)$$

In this case optimal employment entails

$$\frac{\partial \pi_2}{\partial N_2} = \frac{\theta_H}{N_2} - W = 0 \qquad (4)$$

And then

$$N_2 = \frac{\theta_H}{W} \qquad (5)$$

Equation (5) is employment in the second period when productivity turns out to be high: in this case employment does not depends on firing costs *z*.

If we have θ_L (with probability *p*), the firms necessarily want to decrease N_1 and $N_1 - N_2 > 0$. Then second period profits are:

$$\pi_2 = \theta_L \log N_2 - WN_2 - z \cdot \max \quad (N_1 - N_2, 0) = \theta_L \log N_2 - WN_2 - z \cdot (N_1 - N_2) \qquad (6)$$

In this case optimal employment entails

$$\frac{\partial \pi_2}{\partial N_2} = \frac{\theta_L}{N_2} - W - z = 0 \qquad (7)$$

And then

$$N_2 = \frac{\theta_L}{W-z} \qquad (8)$$

Equation (8) is employment in the second period when productivity turns out to be low: in this case employment depends positively on firing costs *z*.

First period decisions

Given (5) and (8) we have that the total expected profits are:

$$E(\pi) = \log N_1 - WN_1 + p\theta_L \log\frac{\theta_L}{W-z} + (1-p)\theta_H \log\frac{\theta_H}{W} - WN_2 - p \cdot z \cdot \left(N_1 - \frac{\theta_L}{W-z}\right) \qquad (9)$$

In the first period, firms choose N_1 optimally to maximise the above expected profits. Then

$$\frac{\partial E(\pi)}{\partial N_1} = \frac{1}{N_1} - W - p \cdot z = 0 \qquad (10)$$

And then

$$N_1 = \frac{1}{W+p\cdot z} \qquad (11)$$

Equation (11) is employment in the first period and it depends negatively on firing costs *z*.

Average expected employment

Average expected employment is $E\left(\frac{N_1+N_2}{2}\right)$. We have that

$$E\left(\frac{N_1+N_2}{2}\right) = \left(\frac{1}{W+p\cdot z} + p \cdot \frac{\theta_L}{W-z} + (1-p)\cdot\frac{\theta_H}{W}\right)/2$$