ADVANCED MACROECONOMIC THEORY

Objectives of Lecture 1

- show how to go from discrete time to continuous time
- become familiar with notation
- formulating the Ramsey model

1.0 Notation

C_t consumption at time t (aggregate/single consumption good)

$F(K_t, L)$ production/output at time t

K_t capital stock

L labour

1.1 Resource Constraint (discrete time)

$$K_{t+1} = K_t + F(K_t, L) - C_t \qquad (1)$$

N.B.

we have ignored
- depreciation
- variations in population size or labour supply

(L fixed)

Also, the resource constraint is in <u>discrete-time</u>

But, we'll begin with <u>continuous-time</u> models

1.2 Resource Constraint in Cont. ~~t~~ time

Rearrange (1)

$$K_{t+1} - K_t = F(K_t, L) - C_t$$

Consider "small" units of time, Δ

$$K_{t+\Delta} - K_t = F(K_t, L) - C_t$$

K_t, C_t etc. are functions of time

Can write them as functions

$$K(t+\Delta) - K(t) = F(K(t), L) - C(t)$$

Divide by Δ

$$\frac{K(t+\Delta) - K(t)}{\Delta} = \frac{F(K(t),L)}{\Delta} - \frac{C(t)}{\Delta}$$

Denote F and C <u>per unit</u> of time

$$C(t) = c(t) \cdot \Delta$$
$$F(K(t), L) = \underbrace{f(K(t), L)}_{\text{production per unit of time}} \cdot \Delta$$

$$\frac{K(t+\Delta) - K(t)}{\Delta} = \frac{f(K(t),L) \cdot \cancel{\Delta}}{\cancel{\Delta}} - \frac{c(t) \cdot \cancel{\Delta}}{\cancel{\Delta}}$$

$$\frac{K(t+\Delta) - K(t)}{\Delta} = f(K(t), L) - C(t)$$

$$\lim_{\Delta \to 0} \frac{K(t+\Delta) - K(t)}{\Delta} = \frac{dK(t)}{dt}$$
$$\left(= K'(t)\right)$$
$$= \dot{K}(t)$$
$$= \dot{K}$$

$\Rightarrow$

$$\boxed{\dot{K}(t) = f(K(t), L) - C(t)} \quad (2)$$

You don't need to remember how to go from discrete time to continuous time. You only have to <u>understand</u> equation (2) [and remember it].

One of the most difficult things in macro is to write the resource constraints right. Take time to understand how to write them!

Utility

- "representative" individual
- discrete time

$$V(c_0, c_1, c_2, \dots$$

- additive separable utility

$$V = u^{0}(c_0) + u^{1}(c_1) + \dots$$

(N.B. c_0, c_1 etc are <u>normal</u> goods)

- <u>discounting</u> & <u>time</u> <u>independence</u>

$$V = \underbrace{\beta^0}_{=1} u(c_0) + \beta^1 u(c_1) + \beta^2 u(c_2) + \dots$$

$0 < \beta < 1$ is discount factor

$\beta =$ discount factor

$$\beta = \frac{1}{1+\rho}$$

then ρ is rate of time preference

= discount rate

Age of consumer?

finite or <u>infinite</u>

$\downarrow$

dynastic interpretation

suppose individuals live for 2 periods but care about their offspring

$$V^0 = u(c_0) + \beta u(c_1) + \beta^2 \underbrace{V^2}_{\text{utility of offspring}}$$

$$V^2 = u(c_2) + \beta u(c_3) + \beta^4 V^4$$

$$V^0 = u(c_0) + \beta u(c_1) + \beta^2 \left[u(c_2) + \beta u(c_3) + \beta^4 V^4 \right]$$

$$= u(c_0) + \beta u(c_1) + \beta^2 u(c_2) + \beta^3 u(c_3) + \beta^6 V^4$$

$$= \sum_{t=0}^{\infty} \beta^t u(c_t)$$

1.3 Utility in Continuous-Time

$$V = \int_0^{\infty} e^{-\rho t} u(c(t))\, dt \qquad (3)$$

c.f. Discrete Time

$$V = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

"an integral instead of a sum"

ρ is discount rate

$e^{-\rho}$ is discount factor

c.f. Discrete Time

let $\beta = \frac{1}{1+\rho}$

ρ discount rate

β discount factor

1.4 Reflections

<u>Why discounting?</u>

Impatience - "consuming early rather than late"

Principle of discounting widely accepted nowadays among economists
(It has not always been so, e.g. Ramsey (1928) argued it to be "immoral")

Discounting at <u>constant</u> rate not always accepted, (i.e. ρ invariant with respect to consumption) Reference: Obstfelt

"Cannot" make ρ function of t only

would cause <u>preferences</u> to be <u>time-inconsistent.</u>

Conceptually and philosofically very difficult.

[Talk about this later in course]

<u>Different from micro?</u>

Only concept of <u>time</u> and aggreg. good

1.5 A Simple Macro Model

(version of which first formulated by Ramsey 1928)

Planner's Problem

(central planning for time being. We'll decentralise later)

$$\max \int_0^{\infty} e^{-\rho t} u(C(t))dt \qquad (3)$$

s.t.

$$\dot{k}(t) = f(K(t), L) - c(t) \qquad (2)$$

This is an <u>optimal-control problem</u>!

We solve it by using a similar method to Lagrange optimisation

[we have no time to go through the foundations, so we just have to accept the <u>rules</u> of optimisation]

1.6 Next Time

Next time we will use optimal control to solve the Ramsey model.

Literature

Heijdra, B.J., and van der Ploeg, F.

"Foundations of Modern Macroeconomics"

pp. 422 - 431 (Ch 14.5 - 14.5.5)

+ pp. 700 - 702 (Appendix 8)

OR

Blanchard and Fischer Ch. 2.

Romer Ch. 2.1 - 2.6