ADVANCED MACRO

Productive public expenditure

Public spending that increases the amount of production (i.e. public spending as a productive input). This could take two forms:

(i) spending that only has a productive effect for one period, and would need to be renewed every period (example spending on police force, legal framework, defense, refuse collection),

(ii) spending that lasts longer, but perhaps depreciates (example spending on infrastructure, roads, railway track, equipment in public services).

For analysis, public infrastructure (ii) is more involving as we would have another state equation, where the level of infrastructure is a state variable and the addition to it is the control variable. It would give us a system of two state equations, plus at least the consumption Euler equation, and be more involving than a graphical analysis. We shall instead in this lecture focus on case (i), public expenditure as a flow (control) variable.

$g_{(t)}$ = amount of public expenditure (lasting at time t only)

Production = $f(k(t), L, g(t))$ and is increasing in $g_{(t)}$

The resource constraint becomes:

$$\dot{k} = f\left(k_{(t)}, L, g_{(t)}\right) - k_{(t)} - g_{(t)}$$

Planner's problem

$$\max_{c_{(t)}k_{(t)}} \int_0^{\infty} e^{-pt}\, u\big(c_{(t)}\big)\mathrm{dt}$$

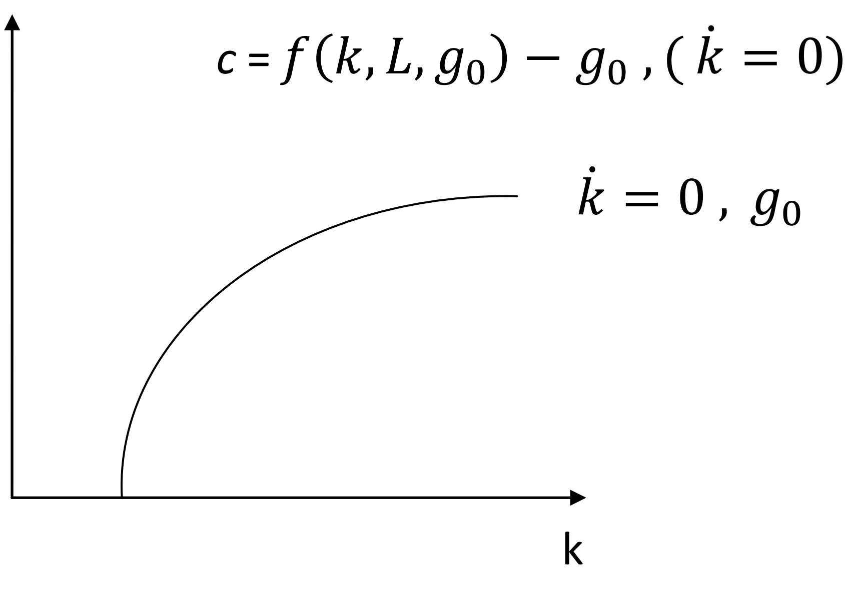
s.t. $\dot{k} = f\big(k_{(t)}, L, g_{(t)}\big) - c_{(t)} - g_{(t)}$

taking $g_{(t)}$ as given $\Rightarrow$ $\dot{c} = \frac{u_c}{-u_{cc}}\ \left[f_k\big(k_{(t)}, L, g_{(t)}\big) - \rho\right]$

Government expenditure affects capital's marginal product. We will assume that it is increasing in *g*

$$\frac{\partial f_k}{\partial g} = f_{kg} > 0$$

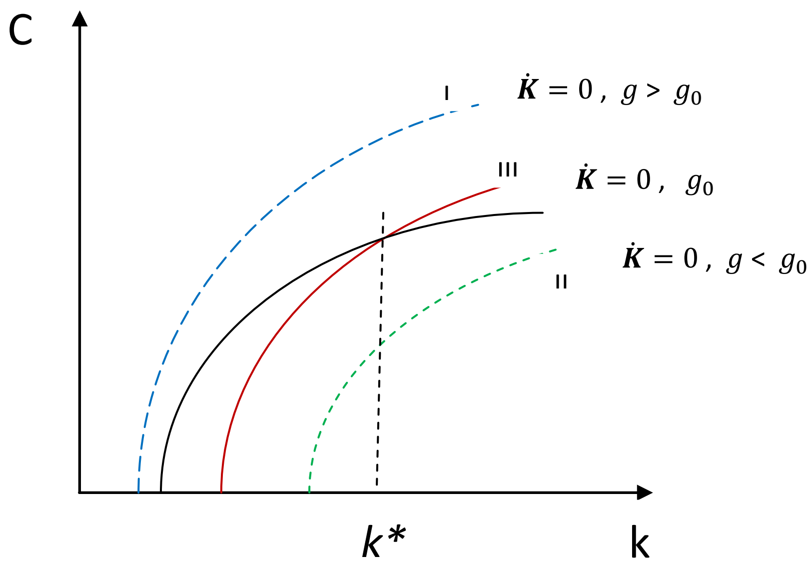
We will now draw the $\dot{k} = 0$ curve (i.e. all combinations of *c* and *k* that make *k* constant, for a given *g*):



What is the effect if g_0 increases?

First, the slope of $\dot{k} = 0$ is $f_k(k, \mathrm{L}, g_0)$ and will increase as g_0 increases. That is, the $\dot{k} = 0$ curve becomes **steeper**.

We then, graphically, have three possibilities when depicting the new $\dot{k} = 0$ curve for the new level $g > g_0$:



In situation (I), an increase in g increases private consumption possibilities for every (graphically depicted) level of *k*.

In this case the original level of g (g_0) was productivity inefficient for all (graphically depicted) levels *k* [too low for all *k*].

In situation (II), an increase in g reduces private consumption possibilities for every (graphically depicted) level of *k*.

Here the original level of g (g_0) was too high for all (graphically depicted) levels of *k* [productivity inefficient].

In case (III), the $\dot{k} = 0$ curve crosses the initial $\dot{k} = 0$ curve at some level of *k*, say *k**. In this case, there is some graphically depicted level of k for which the initial level of g (g_0) is productivity efficient. If the change in *g* was "small", then g_0 was productively efficient for the level of *k* where the pivoting point is (i.e. for *k**).

What do we refer to as production efficiency?

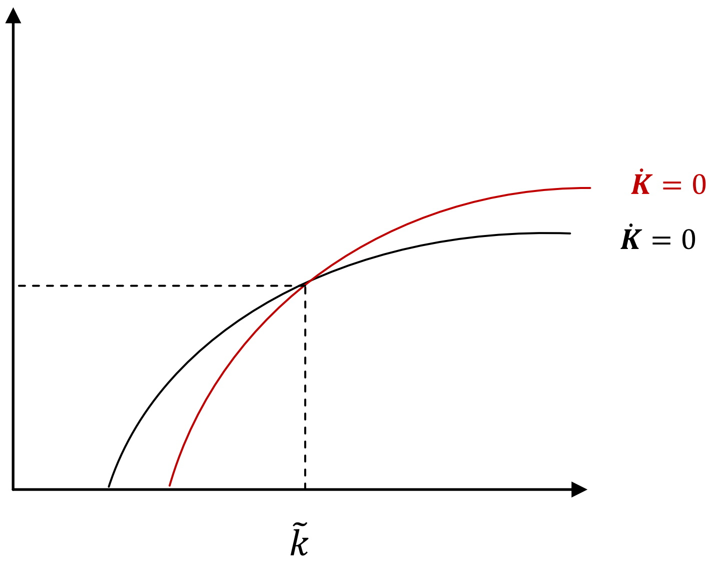
The productively efficient level of *g* is the one that maximizes the consumption possibilities for a particular *k*, that is

$\max_g \quad f(k, L, g) - g \quad \Rightarrow \quad$ FOC $\quad f(k, L, g) - 1 = 0 \quad \Rightarrow \quad g = \emptyset(k)$

Consequently, the productively efficient *g* is a function of *k*, and varies with *k*.

We will concentrate on cases where g is productively efficient for a level of the capital stock not too far from its steady state level (i.e. where the crossing point is not too far away from the steady state capital stock, case (III)).

For "small" changes in g the crossing point is the level of *k* for which g was productivity efficient, say $\tilde{k}$, because when *c* is maximized with respect to *g*, then *c* does not change with a small change in *g* (by construction). We depict this situation in the figure below. The new $\dot{k} = 0$ curve is depicted in red, and is assumed to be arbitrarily close to the initial one (depicted in black).



The productivity efficient level of g is increasing in k. To see this remember that $f_g(k, L, g) - 1 = 0$ when g is productivity efficient for k. To find the relation between k and g that preserve production efficiency, we take the differential of the productive efficiency condition above $\Rightarrow$ $df_g(k, L, g) = 0$

That is

$$df_g = f_{gk}dk + f_{gg}dg = 0$$

$$f_{gg}\frac{dg}{dk} = -f_{gk}$$

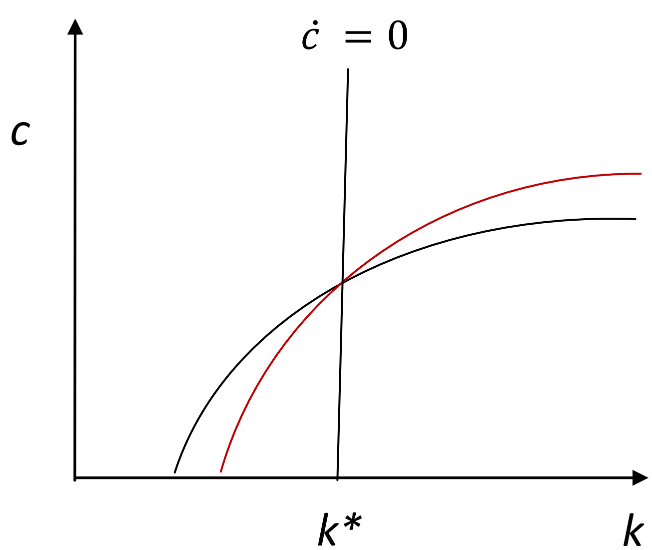
$$\frac{dg}{dk} = -\frac{f_{gk}}{f_{gg}}$$

For $f_{gk} > 0$ and $f_{gg} < 0 \Rightarrow \frac{dg}{dk} > 0$

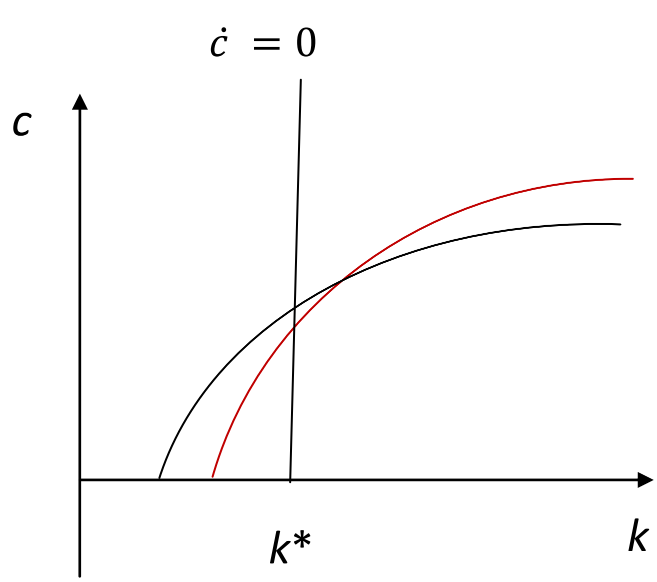
This means that the productive efficiency point for small g is a smaller k, and vice versa. If we get the pivoting point at a small k, it means that a small g was productively efficient, etc. We shall illustrate those situations graphically below.

"Small" changes in g (dg close to zero)

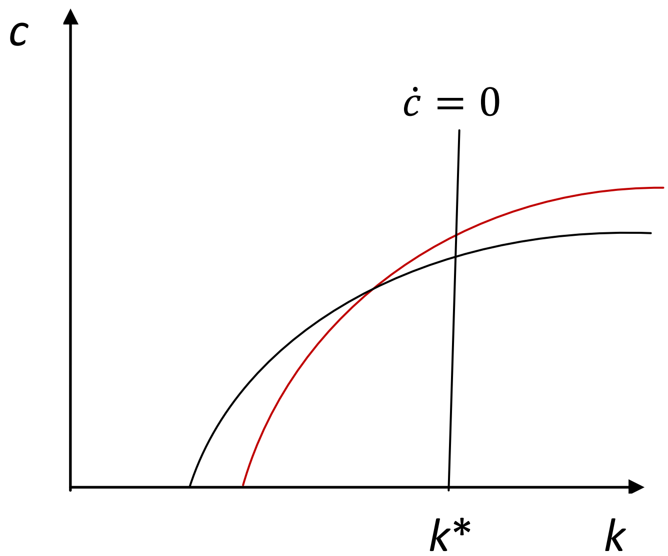
If the initial level of g (g_0) was productivity efficient for the steady-state level of *k* (*k**), we have



If instead the initial level of g (g_0) was too high for *k**, we have



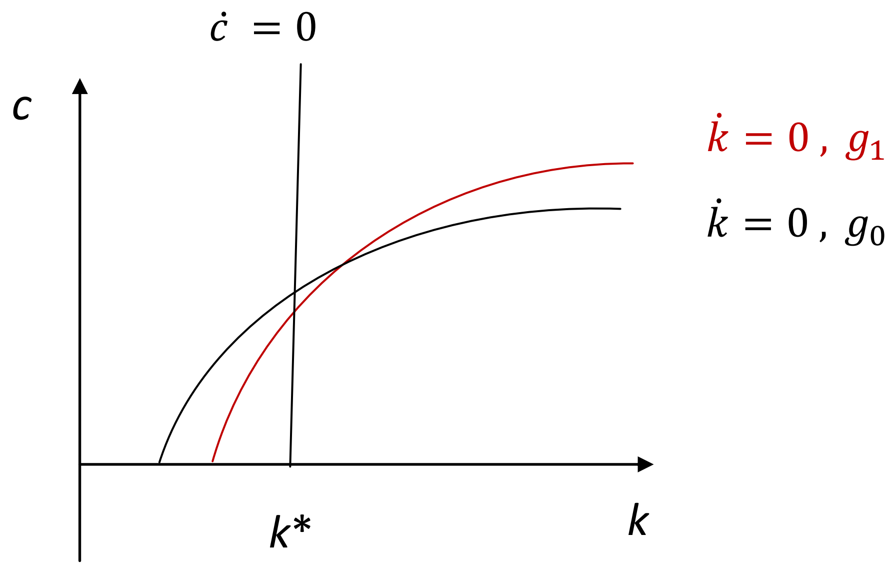
Finally, if the initial level of g (g_0) was too low for *k**, we have



"Large" changes in *g*

Remember that there is a positive relation between the productively efficient level of *g* and the capital stock. This implies that if the original level of *g* was productively efficient and we make a large increase in *g*, as *g* increases to preserve production efficiency, one would have to increase *k* as well. But if *k* is fixed, then, as *g* increases, *k* becomes too low, or alternatively *g* too high. The new crossing point will be for a higher *k* (i.e. the crossing point moves away), shown in the figure below.

If g_0 was productivity efficient for k* increasing it makes it "too large" implying the crossing point moves to the right:



The consumption Euler equation

To analyse the complete phase diagram we need the consumption Euler equation as well. Remember that the $\dot{c} = 0$ line is given by the steady state capital stock such that capital's marginal product is equal to the rate of time preference, i.e.:

$\dot{C} = 0 \quad f_k(k^*, l, , g) = \rho$

How does the steady-state capital stock vary with g?

That is, what is $\frac{dk^*}{dg}$ = ?

We can figure it out intuitively as follows. Since f_k is increasing in g

$g \nearrow \Rightarrow f_k$ greater than ρ need to reduce f_k. Since f_k is falling in k, this is done by increasing $k \Rightarrow \frac{dk^*}{dg} > 0$

Formally, we can show it by taking the differential of capital's marginal product:

$$df_k(k^*, L, , g) = 0$$

$$\underbrace{df_k = f_{kk}\, dk^* + f_{kg}\, dg = 0}$$

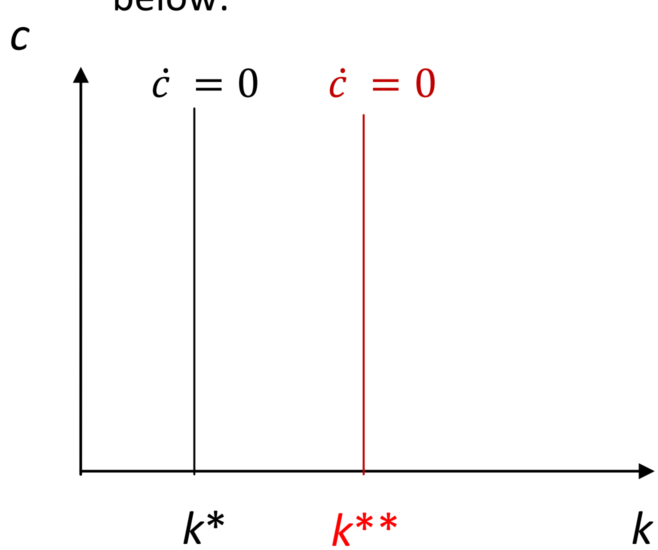
$$\Downarrow$$

$$dk^* = \frac{f_{kg}}{f_{kk}}\, dg$$

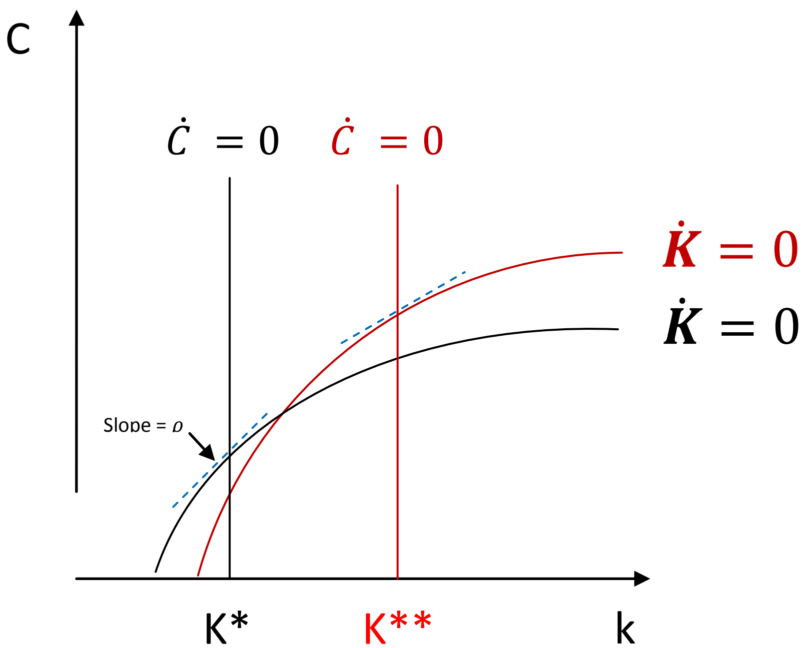
$$\Downarrow$$

$$\frac{dk^*}{dg} = -\frac{f_{kg}}{f_{kk}} > 0$$

Graphically it implies a shift to the right in the cdot line, as shown below:



We are now in a position to draw the phase diagram. Consider the situation where either g initially too high (relatively to its productivity efficient level) or where g was initially productivity efficient for k* but change is "large". This is depicted below:



We will now proceed to analyze the response of an economy due to government changes in g.

As we will see, the response in consumption and investment will depend on whether the initial level was productively efficient or not.

Knowing how the economy responds, we will be able to: either predict a response due to government spending increases or cuts, or, ex post a spending increase or cut, to infer from the responses whether the initial level of g was productively efficient.

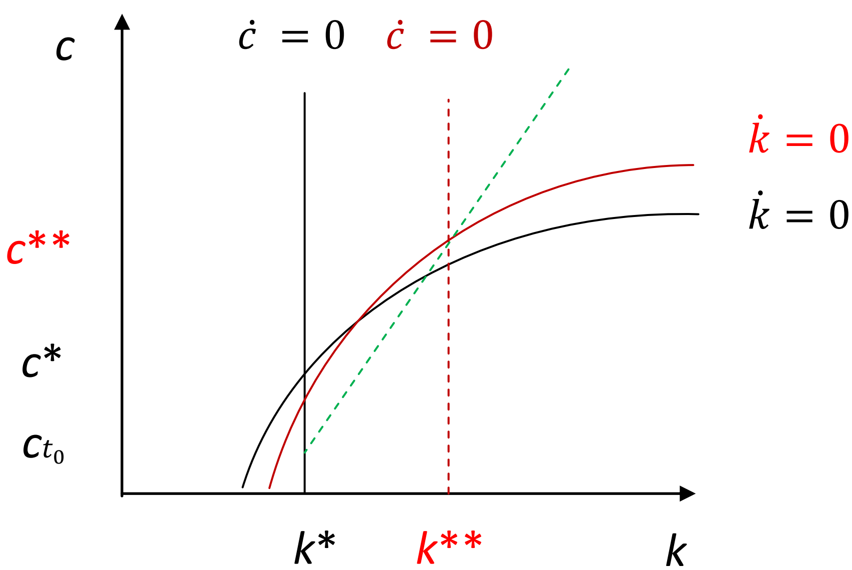
Analysis of policy changes

Suppose initial g was too high relative to its productivity efficient level for k* and there is a surprise increase in g.

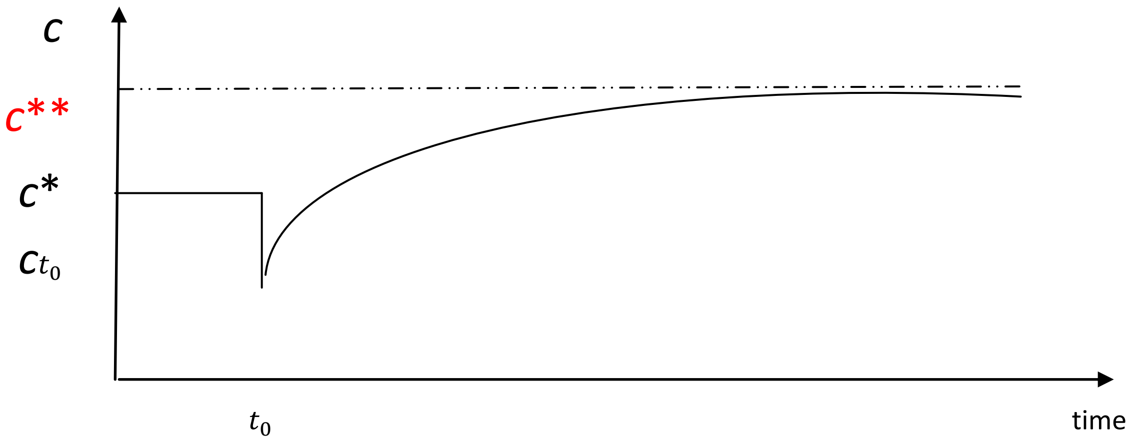
We conduct the analysis the usual way:

1. Draw the new $\dot{k} = 0$ and $\dot{c} = 0$ lines
2. Draw the trajectory in new system
3. Determine how consumption jumps onto the new trajectory

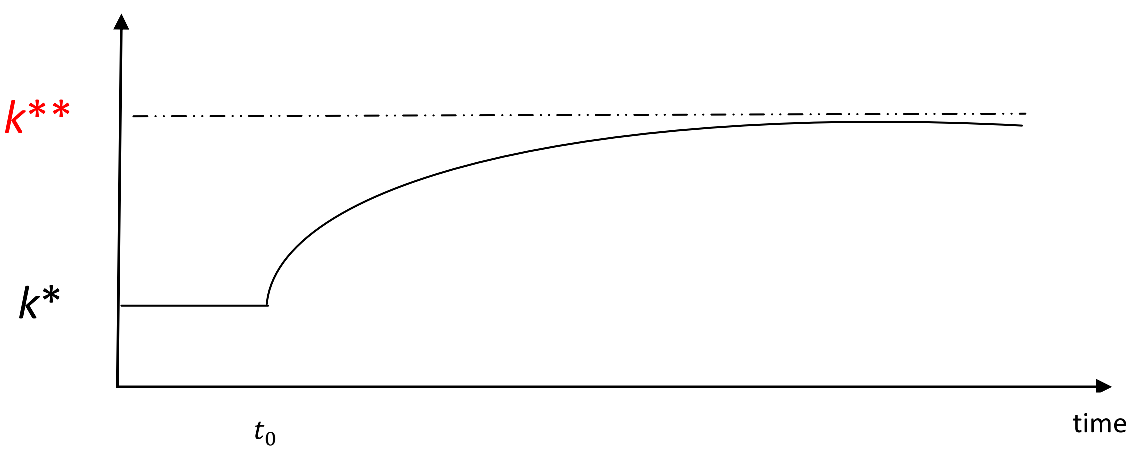
The trajectory in the new system is shown by the green dashed line, below



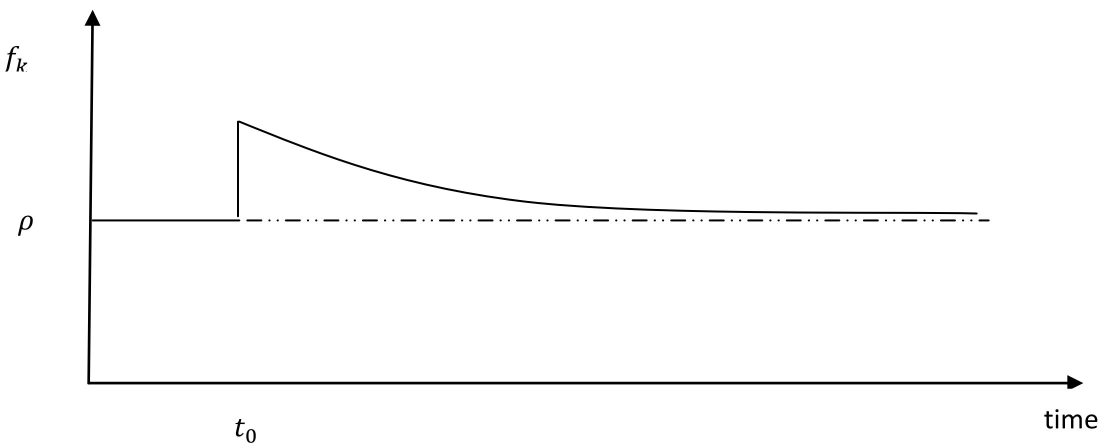
Consequently, c jumps from c* to c_{t0}, then follows the green trajectory toward c**. The time path of c is depicted below.



Remember, *k* cannot jump, but it will gradually increase (along the green trajectory) toward the new steady state level, *k***, as shown below:

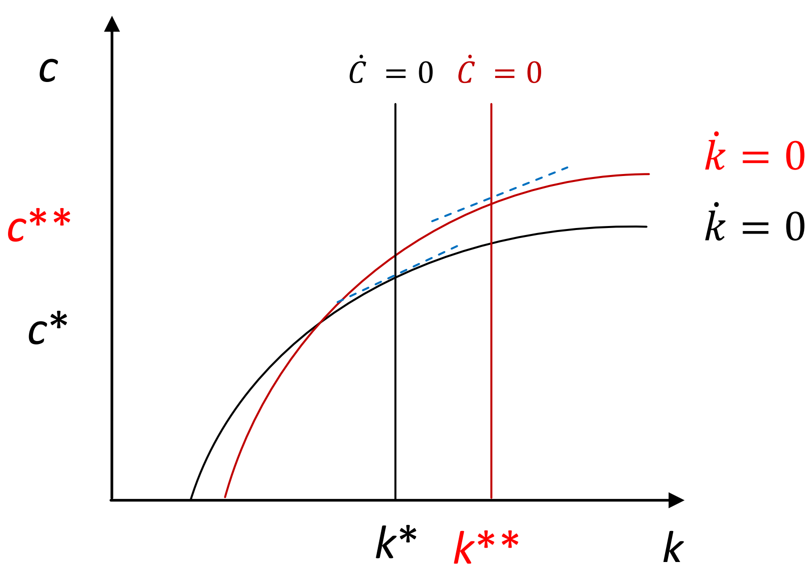


Finally, capital's marginal product (the equilibrium interest rate in a competitive economy) jumps at date t_0 (because of the increase in *g*) and gradually falls back to its original level, ρ, (as capital is increasing). The time path is depicted below.

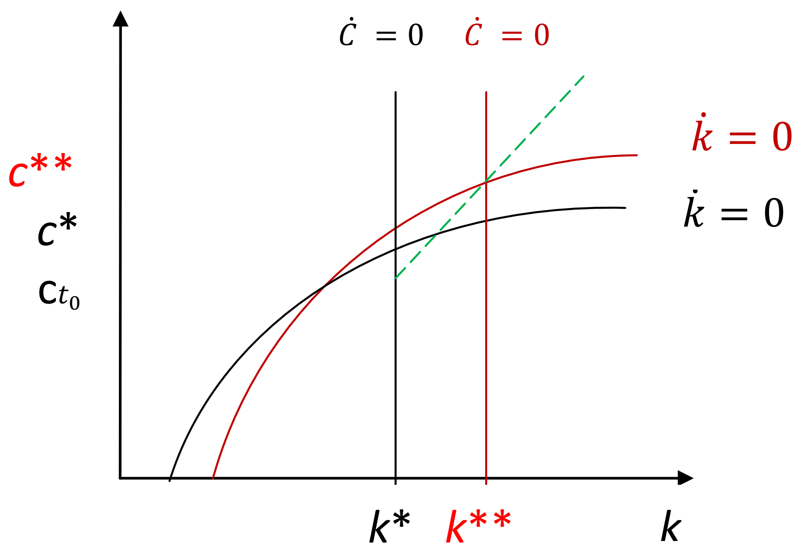


Suppose instead the initial g was too low relative to its productivity efficient level for k* and there is a surprise increase in g.

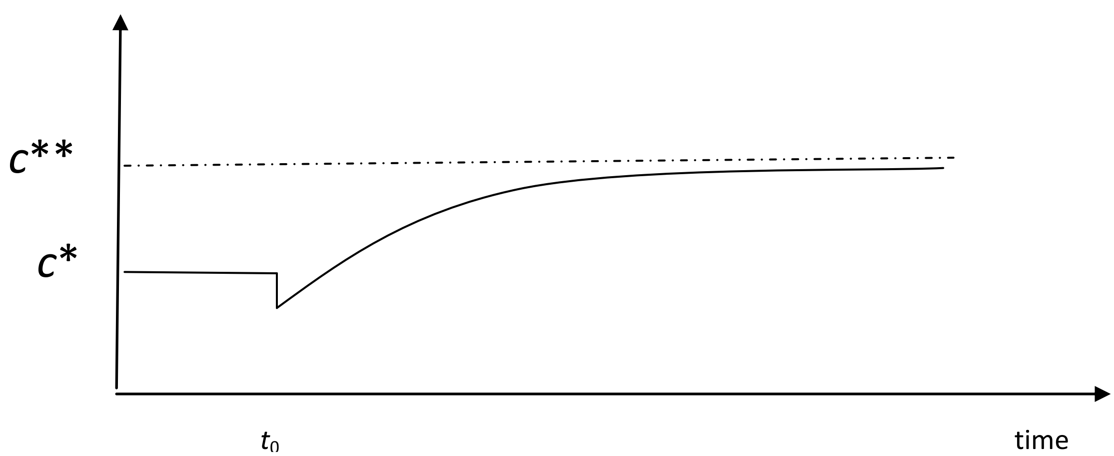
We first draw the new $\dot{k} = 0$ and $\dot{c} = 0$ lines. The pivot point is now left of k*. The dashed lines indicate the slope of the $\dot{k} = 0$ curves at the respective steady states (equal to ρ).

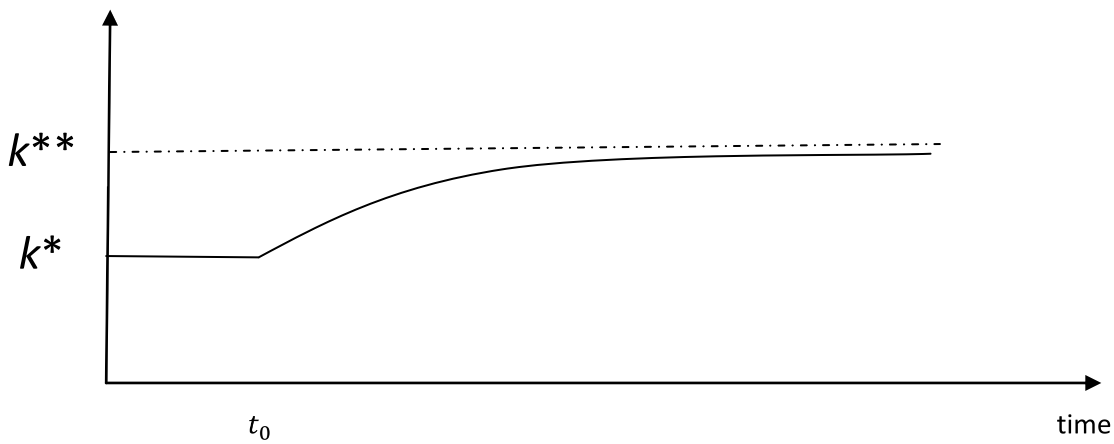


Drawing the trajectory to the new steady state, consumption jumps down from c^* to c_{t0} and follows the trajectory toward c^{**}. Notice that the consumption jump is smaller than in the previous case.



The time paths of consumption and capital are shown below:





Differences in responses between the two economies

If initial level of g is **too high** ⇒ larger cons. jump when $g \nearrow$

If initial level of g is **too low** ⇒ smaller cons. jump when $g \nearrow$

If the government increases g from an already too high level, there is a larger jump in consumption than if the initial level had been too low.

This suggests a way of detecting whether g was too high or low originally. If the initial consumption response due to an increase in g was large, then the original level of g was too high.

Exercise

Consider an economy with productive public spending. Assume the government at date t_0 announces a future increase in public spending, to be implemented at date t_1, and remain constant forever. Analyze the economy's response (in terms of consumption, capital, and capital's marginal product) as a result. Do this for two different economies, one where the initial g is too high and one where it is too low (relative to their productively efficient levels).