

LECTURE 13

PUBLIC GOODS: SAMUELSON RULE

- Aim of Lecture 13:**
- Introduce the idea behind public goods
 - Gain understanding of why government intervention typically is needed in the presence of public goods
 - Derive the Samuelson rule for (first-best) optimal provision under various assumptions on the underlying economy

[Reference for this lecture: Myles (1995), pp.263-269].

13.1 *Public Goods*

13.1.1 *Introduction*

We have so far in the course analyzed taxation (By taxing individuals the government can raise revenue (Ramsey, Diamond Mirrlees, Chamley, Lucas) and redistribute (Mirrlees, Meltzer Richard, Persson and Tabellini). We will now analyze the expenditure side (other than transfers to the individuals). Government expenditure is on Public Goods and Public Inputs (public factors of production, such as infrastructure). In this lecture we will analyze Public Goods and its optimal provision level in the first best (the so called Samuelson Rule).

13.1.2 *Definition of a Public Good*

Just as the name sounds it's a good that can be consumed collectively by more than one individual. Examples are street light, defence, policing, public parks, broadcasting.

We may distinguish between *pure public goods* and *impure public goods*:

Pure public goods

Goods that exhibit *non-excludability* (no household can be excluded from consuming) and *non-rivalry* (consumption by one household does not reduce the quantity available for others)

Impure public goods

Goods where one can either exclude some households from consuming, or where the amount consumed depend on other individuals' consumption.

Exercise 13.1

How would you classify the following public goods:

- street light
- defence
- policing
- public parks
- broadcasting

13.2 A Pure Public-Good Economy

13.2.1 The Economy

For simplicity we will consider 2 households, one private good, x , and one public good, G . For a general treatment (H households, n private goods) see Myles (1995), pp. 266-270.

Utilities

$$U^h = U^h(x^h, G)$$

for $h=1,2$.

Aggregate production constraint

$$F(X, G) \leq 0$$

where $X=x^1+x^2$.

Example 1: Suppose individuals supply labour inelastically. Denote total labour supply L . Suppose production is constant returns to scale in labour, then X and G must be linear in L . So $X=vL^X$, and $G=\theta L^G$, (L^j denotes labour used in sector $j=X,G$). Consequently, since $L=L^X+L^G$,

$$X=v(L-L^G) = v(L-G/\theta)$$

\Leftrightarrow

$$X-v(L-G/\theta) = 0.$$

So in our example $F(X, G) = X -v(L-G/\theta)$.

Example 2: Let the private good be leisure. Suppose production of the public good is constant returns to scale in labour, then G must be linear in total labour supply. Denote the time-endowment individual h , as ω^h . Then G must be linear in $\omega^1+\omega^2-x^1-x^2$. So $X=vL^X$, and $G=\theta L^G$, (L^j denotes labour used in sector $j=X,G$). Consequently, since $L=L^X+L^G$,

$$G = \theta(\omega^1+\omega^2-x^1-x^2) = \theta(\omega^1+\omega^2-X)$$

\Leftrightarrow

$$\theta X + G -\theta(\omega^1+\omega^2)= 0.$$

So in this example $F(X, G) = \theta X + G -\theta(\omega^1+\omega^2)$.

Exercise 13.2

Suppose the individuals can buy G on the market. Show the budget constraints of the two individuals for Example 2 above, and formulate the utility maximisation problem.

13.2.2 The Samuelson Rule

We wish to find all first-best allocations. We solve the Pareto problem: Maximising utility of one individual subject to the utility levels of the others not falling below a certain target level, U^h . By varying the target levels of utilities one can trace out the entire Pareto frontier.

The Pareto problem is to

$$\begin{aligned} \max \quad & U^1(x^1, G) \\ \text{s.t.} \quad & U^2(x^1, G) \geq \bar{U}^2 \\ & F(x^1 + x^2, G) \leq 0 \end{aligned}$$

Lagrangian:

$$L = U^1(x^1, G) + \mu^2 \{U^2(x^1, G) - \bar{U}^2\} - \lambda F(x^1 + x^2, G)$$

First-order conditions:

$$U_x^1 - \lambda F_x = 0 \tag{i}$$

$$\mu^2 U_x^2 - \lambda F_x = 0 \tag{ii}$$

$$U_G^1 + \mu^2 U_G^2 - \lambda F_G = 0 \tag{iii}$$

Try to substitute away the multipliers. Use equation (ii) in equation (iii) to eliminate μ :

$$U_G^1 + \lambda F_x U_G^2 / U_x^2 = \lambda F_G$$

Divide through by U_x^1 and use equation (i) to eliminate λ :

$$U_G^1 / U_x^1 + U_G^2 / U_x^2 = F_G / F_x \tag{Samuelson Rule}$$

This is the Samuelson Rule. It says that the sum of marginal rates of substitutions (between the public good and the private) should equal the marginal rate of transformation between the two goods.

13.3 Next Time

Next time we will introduce a price system (Lindahl Pricing) that is able to decentralise public-goods provision and implement the first-best allocation.