VOLUNTARY PUBLIC GOODS PROVISION: APPLICATION TO CLUBS

Consider the voluntary public goods model, but where individuals form clubs ("Big Societies"). For simplicity consider two clubs. There are N individuals in club 1 (N' in club 2). The contribution of club 1 to the public good is G (and that of club 2 G'). Each member of the club pays a lump-sum payment. There is one decision maker in each club, whose endowments are ω and ω' in club 1 and club 2, respectively.

The decision maker in club 1 chooses G, taking G' as given (Nash equilibrium), so as to maximise

$$U = u(\omega - G/N) + v(G + G')\,. \tag{1}$$

The problem of the decision maker in club 2 is to maximise (just swapping no $'$ for $'$, etc)

$$U' = u(\omega' - G'/N') + v(G' + G)\,. \tag{1'}$$

The first-order conditions are:

$$-u'(\omega - G/N)/N + v'(G + G') = 0\,, \tag{2}$$

$$-u'(\omega' - G'/N')/N' + v'(G' + G) = 0\,. \tag{2'}$$

These two equations can be solved to yield G and G' as solutions (as functions of ω, ω', N, N'). Before solving for an example, we shall explore some general results.

In general, if both private and public consumption are normal goods, G tends to be increasing in ω, declining in ω', increasing in N, declining in N'. Everything else equal, the club with the poorer decision maker provides less and the other club more. The more numerous club provides more and the other less.

The role of the wealth of the decision maker

Take an arbitrary individual in club 1, with an endowment of ω^i (not necessarily equal to the endowment of the decisive individual, ω). Her utility function is

$$U = u(\omega^i - G/N) + v(G + G')\,. \tag{3}$$

Suppose individual i can chose the decisive individual (through an election or by simply appointing) prior to the public goods contribution game. Taking the derivative of the utility function with respect to the identity (i.e. wealth) of the decision maker, realising that G and G' are functions of ω, we get

$$\frac{dU^i}{d\omega} = \left[-u'(\omega^i - G/N)/N + v'(G + G')\right]\frac{dG}{d\omega} + v'(G + G')\,\frac{dG'}{d\omega}\,. \tag{4}$$

Evaluating this derivative at $\omega^i = \omega$ (implying the term in square brackets is zero, by equation (2)), we get

$$\frac{dU^i}{d\omega} = v'(G+G')\frac{dG'}{d\omega}. \tag{5}$$

Thus, if the other club's provision is declining in ω (i.e. if $dG'/d\omega < 0$) then the individual's utility is declining in ω when evaluated at the persons own identity. Consequently the individual would like to appoint someone poorer than herself, to take the public good decision, in the game with the other club. The reason is, that by strategically choosing a poor decision maker, club 1 strategically commits to provide less of the public good, and club 2 in equilibrium is partially making up for this by providing more. Club 1 is thus free riding on club 2. The total amount of the public good will then be lower (especially if club 2 is thinking in the same way).

To conclude, both clubs have an incentive to appoint a poorer decision maker in order to free ride, and the equilibrium level of public goods will be lower. If there are self organising clubs, they will tend to behave as if they are poor clubs, providing less in equilibrium.

Club size

What are the incentives regarding membership? Due to the nature of public goods one ought to have large clubs. If clubs can decide membership, how large would they be? For simplicity, we ignore any differences in ω within the club, and focus on membership size.

Differentiating the utility function of the decisive indivicual (equation (1)) with respect to *N*, realising that *G* and *G′* are functions of *N*, we get

$$\frac{dU}{dN} = \left[-u'(\omega - G/N)/N + v'(G+G')\right]\frac{dG}{dN} + u'(\omega - G/N)\frac{G}{N^2} + v'(G+G')\frac{dG'}{dN}$$

$$= v'(G+G')\left(\frac{G}{N} + \frac{dG'}{dN}\right) \tag{6}$$

where the second equality follows from (2).

Thus, if the other club's provision is reduced by more than G/N, when *N* (is increased (i.e. if $dG'/dN \leq -G/N$) then the individual's utility is declining in club size. Consequently, each club has an incentive to become as small as possible. This is again due to the free riding problem. Big societies can have the tendency to become small societies.

Exercise - An example

Consider the logarithmic utility function, i.e.

$$U = \ln(\omega - G / N) + \ln(G + G'). \tag{7}$$

$$U' = \ln(\omega' - G' / N') + \ln(G' + G). \tag{7'}$$

(a) Show that the Nash equilibrium contribution levels are:

$$G = \frac{2\omega N - \omega' N'}{3} \tag{8}$$

$$G' = \frac{2\omega' N' - \omega N}{3} \tag{8'}$$

(b) Show that the indirect utility of a member, i, of club 1 is:

$$U^i = \ln\left(\omega^i - \frac{2\omega N - \omega' N'}{3N}\right) + \ln\left(\frac{\omega N + \omega' N'}{3}\right) \tag{9}$$

(c) Show that utility is decreasing in ω (when ω^i is close to ω). What is your conclusion?

(d) For $\omega^i = \omega$ and total population being fixed, $\overline{N} = N + N'$, show that the indirect utility of a club 1 member can be written as

$$U = \ln\left(\omega - \omega' + \omega' \frac{\overline{N}}{N}\right) + \text{constant} \tag{10}$$

and that it is declining in membership. What is the 'optimal' club size?